Integrated Temporal-Spatial Model for Construction Plans with Boolean Logic Operators

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Abstract

A novel construction project planning model is presented that allows more efficient and realistic planning than current methods. Unlike scheduling methods such as CPM, the proposed model represents spatial entities on the construction site such as work areas and safety buffers. The model uses Boolean operators to allow the codification of the actual constraints that guide construction projects, and to create a single integrated temporal-spatial model. Singularity functions are used for the mathematical definition of the constructs and their relationships, providing a flexible modelling framework. The model is implemented in a real-world case study, with input data from BIM, demonstrating its feasibility and usefulness.

Keywords: Construction scheduling; Building Information Modeling; Work paths; Work areas; Spatial relationships; Boolean logical operators.

Introduction

Over the past few decades, numerous authors have leveled critiques against the existing planning methods that are available to construction managers (Senior 2007, Koskela 1992, Birrell 1987). Gantt (1913, p. 130) already wrote (although in a context of production planning): “Many shops have a very nice schedule system; they plan their work beautifully – at least, it looks very pretty on paper; but they have no means of finding out whether those schedules are lived up to or not. Usually they are not.” Six decades later Docherty (1972) echoed this in commenting that it is quite common to find the formal plans prepared at the firm/project level “decorating the project
management office walls on site”, while “execution proper is governed by informal short-term planning performed by site/work field personnel, at times totally disallowing the formal plans.” Laufer and Tucker (1987) similarly noted that formal plans in practice mostly serve for forecasting and retrospective purposes (including litigation) of middle managers, but execution is planned by the on-site management team with more informal methods. Zack (1992) observed that contractors often use schedules to build claims, rather than to plan and coordinate. Recent studies have confirmed these observations, such as Koskela et al. (2014), who noted that the goal of construction planning has shifted to contract control, and Grau et al. (2014), who stated that planning is based on contractual information, but it mostly ignores actual execution constraints.

**Shortcomings of Network Scheduling**

Most of the criticism of planning practices in construction projects has focused on limitations of the critical path method (CPM) that is most commonly used for these purposes (Howell et al. 2010). Birrell (1987) noted that CPM fails because it is based on discrete activities that are connected only at their start and finish points. It ignores continuous and simultaneous spatial work flows of crews on real sites, and the division of such sites into work areas, which in practice guides the planning of deliverables and resource allocation. Birrell (1987) contended that CPM creates less efficient processes than the heuristics that practitioners use, because it cannot model work flows, and thus fails to ensure work continuity for crews. Koskela (1992) concurred that CPM fails to reflect both crew and material flows, and gives suboptimal results.

Construction processes differ from most production processes in that several subcontractors may move on a construction site simultaneously while performing their interdependent activities, in contrast with production lines in which a product moves from one workstation or procedure to the next (Kovács and Kot 2017). The coordination of activities and their interfaces in both space
and time are therefore vital for construction management. This has been recognized in studies in which “4D” scheduling representations were developed (Hamledari et al. 2017). Such representations associate components in the design with activities in the schedule, but are constrained by the fact that spatial and temporal relations and constraints are still represented in two distinct models that are not truly integrated. Any change in one model will directly take into account only one type of relation – temporal relations in the schedule, and spatial relations in the design. Consequently, an improvement in the project’s plan will only be achieved through an iterative process, in which the planner moves from one model to the other, without being able to guarantee that an optimal solution is reached. An integrated model that includes both the temporal and spatial aspects of a construction project would allow planners to directly take into account the various constraints and variables relevant to the project plan, and identify optimal solutions. This requires, however, the definition of a mathematical model that is sufficiently rigorous to allow analysis and optimization, and flexible enough so that it can be adjusted according to the specific requirements of each project. Furthermore, as the following literature review explains, once one represents space in the project model, sequential temporal relations alone are no longer sufficient. Concurrent activities are prevalent on construction sites, and new types of relations need to be defined in order to represent the constraints that preoccupy site managers, and ensure that these activities are carried out safely and efficiently (Isaac et al. 2017).

For these reasons this research creates a new integrated project planning model with spatial constructs and new relationship operators that better reflects the needs of construction managers and facilitates execution. It also lays a foundation for developing future analytical methods.

**Literature Review**

The definition of a new integrated scheduling model considers prior studies in several domains...
that have used relationship types between activities; logical operators to define them; scheduling methods that consider both time and location; and the definition of discrete work areas.

**Relationship Types between Activities**

The network model that currently underlies the scheduling of activities, the precedence diagram, defines precedence relationships among activities. It can be traced back to Salveson (1955), who modeled the assembly line balancing problem. Kelley and Walker (1959) and Fondahl (1962, 1987) used it to support CPM calculations. Since then it has remained largely unchanged, despite numerous critiques of planning methods that rely on it (Hajdu 2015). The network model of CPM and other methods contains only point-wise relationships between activity ends to mark their start and finish dates. As Fondahl’s (1962) report explained, such relationships cannot represent overlapping activities, which are common in practice. His suggestion for this issue was to further divide them into sub-activities. But such fragmentation creates practical problems; first a significant increase in the number of starts and finishes, and second a multiplicative growth in the number of possible relationships between them. Recent studies indicate that a fragmentation may not be required, as redefined point-to-point relationships could connect internal points of activity pairs, which would allow managing as many points as needed (Hajdu et al. 2017, Hajdu 2015, Francis and Miresco 2002). Point-to-point relationships can be defined between any points on predecessor and successor, including a time lag between them. Such internal points can be defined by a period from the activity start point (Hajdu 2015), akin to a time lag after the start.

**Logical Operators**

The current network model implicitly uses only one type of operator, a material implication ‘IF-THEN’. In practice, other logical operators can also be included in heuristic form by construction managers (although they may be unlikely to define them in such formal terms). For example,
executing two activities in the same location may be forbidden (an exclusive disjunction ‘XOR’), or conversely may be required (a logic conjunction ‘AND’). One barely remembered method that explicitly included such operators was the Graphic Evaluation and Review Technique (GERTE). It was created to analyze stochastic directional activity networks (Pritsker and Happ 1966, p. 268) and “combined PERT [i.e. precedence] type networks with flowgraph concepts.” Yet it used the older activity-on-arrow notation (Kelley and Walker 1989) that represents activities as arrows from start to finish nodes. Branches, which model alternative activities, carried probabilities of taking a possible route and their respective durations (Pritsker 1966). Nodes had logic operator symbols as Elmaghraby (1964) introduced: XOR (true if exactly one predecessor occurs), OR (true if one or multiple predecessors occur), and AND (true if exactly all predecessors occur).

However, applications of GERT have been rare in theory and practice (León et al. 2013). One reason is that while GERT can model a given process architecture, it cannot identify how to rearrange activities to improve project performance (Browning and Eppinger 2002). Moreover, as the complexity of the network increases, it becomes increasingly difficult to apply the method, so that simulation becomes necessary (Taylor and Moore 1980). The need for simulation, in turn, makes it difficult to understand the interaction of network structure and project duration (Smith and Eppinger 1997). Similarly, the time, cost, and effort required to acquire competency in simulation languages and software have strongly limited current use of discrete event simulation in practice (AbouRizk 2010); an impressive body of studies with products like STROBOSCOPE (Martínez 1996) or SIMPHONY (AbouRizk and Mohamed 2000) notwithstanding. Similar to GERT, they offer logical operators as constraints for the flows of resources that are consumed or produced. They thus appear more suitable to analyze a given process than to develop a schedule.

**Location-Based Scheduling**
Network models contain only data on temporal aspects, but ignore the intensely space-dependent nature of construction projects completely. This resembles a composer who provides a musical score that displays only the durations of notes for each of the different instruments, but not their actual pitch. Obviously, it would be impossible for any orchestra to perform such a composition.

Location-based scheduling that takes into account both the timing and location of work exists under various names, e.g. linear (Mahdi 2004), repetitive (Harris and Ioannou 1998), or line-of-balance scheduling (Kenley and Seppänen 2010). These methods use a two-dimensional diagram to visually track the progress of work from one location to another over time (Lumsden 1968). Locations themselves, which are coordinates on one axis, are either physical sections within a repetitive facility (e.g. zones, floors, or apartments) or distances along a linear infrastructure project (e.g. a road or railroad). The other – often horizontal – axis indicates time, over which the location is displayed where an activity is executed at a given time. These diagrams are limited in that their axis can only track either a physical division of the site for one type of repetitive unit (for buildings), or a single path across the site (for linear projects). But the actual use of space on a construction site is considerably more complex than that. Different activity types will typically progress along different paths on the site (Riley and Sanvido 1995), and even in infrastructure projects the movements of heavy equipment are not necessarily restricted to a single linear path.

Existing location-based scheduling methods consequently remain limited in representing the locations of activities while planning and controlling time. Progress in overcoming these issues was made by Dagan and Isaac (2015, p. 64), who developed “a matrix-based method for the definition of minimum safe distances between workers, and the use of 3D [three-dimensional] time-space diagrams to represent and analyze the dynamic movements of workers on site.” But
their 3D chart lacked a mathematical formulation of the activities and could therefore not be extended to include other dimensions such as cost, or to compose a scheduling algorithm.

**4D Scheduling**

Following the introduction of 3D object-oriented Building Information Modeling (BIM) for project design, solutions have been developed to associate components in the design with the planned activities for their construction. These “4D” models can be used to support scheduling by automatically calculating quantities and thus durations, and by visualizing construction sequences and thus identifying workspace conflicts (e.g. Sugimoto et al. 2016; Kim et al. 2013).

Yet current 4D models do not support a full representation of either spatial information in the schedule, or of temporal information in the design (Dang and Bargstädt 2015). The fact that these two domains are not integrated prevents spatial information from being directly and fully taken into account in schedules, and as a result prevents truly optimal solutions from being identified. The frequent updates of the schedule that are required following changes in the project can also only be carried out in an efficient and effective way when all dimensions – spatial and temporal – are represented (Son et al. 2017). Currently, the implications of changes in one domain cannot be directly addressed in the other domain (Dang and Bargstädt 2015).

**Work-Space Planning**

A number of studies have addressed the definition of discrete work areas for planned activities per their requirements and corresponding components in the design. Shaked and Warszawski (1995) defined a knowledge-based expert system wherein work areas were captured. Riley and Sanvido (1997) presented a manual space planning method for multistory buildings. It allowed planners to identify potential spatial conflicts to minimize congestion and interference. It defined the work areas that would be required for activities, their locations, and the order in which they
move to them. Akinci et al. (2002) developed a system that formalized the analysis of time-space conflicts. Spatial conflicts were automatically detected in a 4D Computer Aided Design (CAD) model. Winch and North (2006, p. 473) proposed a decision-support tool “for marking up available space, allocating tasks to spaces, and analyzing and optimizing space loading in relation to the critical path”.

None of these studies incorporated a definition of their work areas at the activity-level. This limited their application to retrospective identification and resolution of space conflicts, after the activities had been scheduled, rather than a proactive joint planning of both activity location and timing to simultaneously fulfill their spatial and temporal constraints.

**Summary**

This literature review confirms that existing scheduling methods suffer from several limitations that limit their practical utility by being unrealistically simplistic for spatial aspects per Table 1.

<INSERT TABLE 1 HERE>

**Methodology**

This research hypothesizes that a multi-dimensional mathematical model can be developed that will enable the detailed execution planning of construction activities in their temporal and spatial environment. By directly and quantitatively representing work areas on the site and their spatial relations, as well as planned activities and their temporal relations, the implications of planning decisions in both time and space can be simultaneously analyzed to identify safer and more efficient solutions, while the actual constraints that guide the building process are considered.

The proposed model supports an analysis of the *time-space-criticality* of activities. Whereas time-criticality alone identifies activities for which an insufficient time will affect the completion of successors or the entire project, a time-space-criticality analysis with the proposed model
broadens such insights by identifying both temporal and spatial resources that critical activities require. If those resources are not allocated, this will create time-space conflicts between the activities and hamper a successful project completion. There may, for example, be sufficient time to execute a critical activity, but if its workspace is occupied by another activity at the time at which it should be executed, this will have an adverse impact on the project.

**Research Objectives**

The research seeks to derive rigorous mathematical formulations of constructs for a comprehensive integrated project planning model. Further techniques to improve the project plan (e.g. ‘crashing’ a schedule in a time-cost tradeoff) that require heuristics or optimization can only be implemented if such an integrated model contains explicit mathematical descriptions of all relevant components and their interactions. To accomplish this goal, four objectives are set:

1. To use BIM data to automatically define a project activity network model that explicitly incorporates such temporal-spatial constructs and constraints.
2. To create a mathematical definition of the fundamental spatial entities on the construction site – including work-paths, work areas, and the buffers that are required between them.
3. To create a mathematical definition of both spatial and temporal point-to-point relationships that are used to connect activity pairs. This will facilitate a better planning and control of overlapping activities, which dominate real-world execution.
4. To create a mathematical definition of relationships with different types of logical operators, to represent more realistic constraints within the relationships between activities.

The new approach will be implemented in MATLAB computer code and validated with a real-world case study of a representative construction project that exhibits various logical constraints.

**Assumptions, Scope, and Limitations**
This paper follows an initial study (Isaac et al. 2017), that had used singularity functions for the definition of work paths in a 3D coordinate system. As Table 2 lists, the present study adds vital components to the limited theoretical model of Isaac et al. (2017) so that it becomes capable of expressing planner considerations for spatially complex project such as a high-rise building.

<INSERT TABLE 2 HERE>

The goal of this research is to provide solutions for projects that are spatially more complex than linear ones, e.g. highways, and for which existing location-based scheduling is inadequate. The new approach is thus tested by applying it to a high-rise building. It establishes an integrated model that better reflects the considerations of planning the execution of construction activities and their subsequent control. It will be extensible to allow applications such as time-cost tradeoff analysis and an investigation of its performance with different heuristics and optimization methods. But these are beyond the scope of this theory-building, model-developing research. It is assumed that all constructs can be measured with quantitative units and that known dependencies exist between these dimensions. Beyond this scope, future research will address how to also properly handle temporary resources on site, e.g. cranes, material stockpiles, and scaffolding.

**Model Development**

The proposed model will carry out a temporal-spatial planning process in four steps (Figure 1):

1. Attributes of each activity are defined, including its duration, start and finish constraints, and its spatial coordinates and requirements. As will be explained in the following section, BIM data can be used to efficiently and automatically define activity attributes.

2. Temporal and spatial relationships between activities are defined with Boolean operators.

3. Violations of the relationships are automatically detected through the calculation of
horizontal and vertical distances between work paths and work areas in time-space charts. Conflicts are solved by adding time, space, or resources.

4. Horizontal and vertical distances between work paths and work areas are minimized, subject to relationships and buffers between them, thus improving efficiency of time and space use.

This section mathematically defines the model, including the spatial constructs, temporal and spatial point-to-point relationships, and logical operators that it contains.

**<INSERT FIGURE 1 HERE>**

**Definition of Activity Attributes Using BIM Data**

To define the temporal and spatial attributes of the planned activities, the necessary data on work areas and resource requirements are systematically extracted in an automated manner from a computerized BIM model, such as that shown in Figure 2. This fulfills Objective 1:

1. **Identification of separate work areas in BIM**: A grid of reference points is defined in the model and a ‘level’ attribute is defined for each building floor. The work areas are identified via reference points and levels as ‘phases’ in a BIM software tool, e.g. Revit by Autodesk.

2. **Definition of components in each work area**: In BIM one can either define a new component for an existing work area or assign an existing component to a newly defined work area.

3. **Exporting the list of work areas and components**: A list of components is created for each work area, including its types of components and their sizing. This is done via the material takeoff tool in BIM. It generates a bill of quantities that is organized by the given work areas.

4. **Resource assignment**: Resources that are required for the activities in each work area are defined for the components that are included in the activities. Activity durations in each work area are defined by their resource requirements by performing a time and cost analysis.

5. **Creating the activity network model**: From data on the work areas and the activity durations
within them, a network model is generated by adding relationships to link said activities.

**Spatial Constructs**

The model includes information on the locations of activities in three spatial dimensions, in addition to the temporal attributes of the activities (such as their start date and duration). The model is therefore four-dimensional, but 3D time-space charts are used to visualize how workers and equipment use the work space on site over time to execute the activities (Figure 3). These charts can represent two spatial dimensions and the temporal dimension of an activity, where the horizontal axes of the chart represent coordinates on site, and the vertical axis measures time (Figure 3b). Alternatively, the chart can also visualize all three spatial dimensions (Figure 3a).

The proposed model can represent different types of spatial constructs. *Work areas* describe bounded spaces occupied for the duration of an activity by workers, equipment, and materials (for example when installing floor tiles in a space). Crews are assumed to be located anywhere within the work area for a certain period of time, and the area will accordingly be represented as a prism in the time-space chart (Figure 4c). A work area will usually not include all paths along which crews may move to reach the area (for example from an elevator).

*Work paths* describe the succession of locations of a crew or equipment that performs an activity (for example when installing a Heating, ventilation and air conditioning (HVAC) duct). The model will contain representations of the primary work paths that are used intensively during the execution of an activity, while secondary paths that are rarely or briefly used and that are unlikely to interfere with other activities need not be explicitly modeled. Work paths are
represented as a line in the time-space chart (Figure 4a). If a work path carries by safety buffer to
prevent excessive proximity to other activities, it is shown as an inclined plane (Figure 4b).

Generally, work areas are likely to be more common in construction project planning than
work paths. However, the mathematical definition of a work area in the model (i.e. a prism in the
time-space chart) is based on the definition of a path with safety buffer (i.e. a plane), whose
definition is in turn based on that of a work path (i.e. a line). Therefore, the following
mathematical definition of the spatial constructs in the model will start by describing work paths.

**Definition of Work Paths**

The new approach uses singularity functions – mathematical expressions that are sufficiently
flexible to allow modeling the constructs for this research, or any higher order terms (Lucko *et
al.* 2014). They contain straightforward constant and linear segmental terms, which in the
authors’ experience can be easily implemented in any programming language, yet still checked
with paper and pencil. By using the same type of terms for all constructs in the extended model
(activities, relationships, and spatial entities), an integrated modeling and analysis framework is
gained. It is extensible to further methods that may optimize the project plan by given objectives.

Equation 1 is a basic singularity function that is defined in the point-scalar form, where point
and scalar are *a* and *s*. Its pointed brackets conduct a calculation as a piecewise function: if the
independent variable *x* is smaller than the cutoff value *a*, then the function *y(x)* remains off and is
zero. Otherwise, the function switches on, the pointed brackets are evaluated like round ones,
and the term is calculated with standard algebra. The exponent *n* controls the functions being
constant (*n* = 0), growing linearly (*n* = 1), or nonlinearly (*n* ≠ 1). Singularity functions can be
added, subtracted, multiplied, and divided. In particular, by subtracting two terms with different
cutoffs, one can model a segmental profile between these two points, which is useful in modeling
activities that feature a specific start and finish date and location within construction schedules.

\[ y(x) = s \cdot (x-a) \]

\[ = \begin{cases} 
0 & \text{for } x < a \\
\frac{1}{s} \cdot (x-a) & \text{for } x \geq a 
\end{cases} \quad (1) \]

Following vector calculus, a body moving in space travels on a path that can be represented with the parameter \( t \) in Equation 2, where the variables \( x, y, \) and \( z \) are 3D coordinates (Kreyszig 2011). By multiplying the variables with unit vectors \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k}, \) any point on that path is simply a combination of three coordinates. By further treating \( x, y, \) and \( z \) as functions of \( t \) (representing time, but could also be another managerial variable, e.g. cost, which is beyond the scope of this research and will be addressed in future work), the path vector \( \mathbf{r} \) becomes a function of \( t. \) Thus \( \mathbf{r}(t) \) is the coordinate functions of \( t \) growing in all dimensions. Its components \( x(t), y(t), \) and \( z(t) \) are newly defined as singularity functions. Equation 3 is the point-vector form of a line segment in space as the sum product of start and finish cutoffs of \( t \) (\( t_s \) and \( t_F \)), start point coordinates \((a_x, a_y, a_z), \) and a growth vector \((s_x, s_y, s_z)\) for changes in the dimensions \( s_x, s_y, \) and \( s_z, \) respectively.

\[ \mathbf{r}(t) = [x(t), y(t), z(t)] = x(t) \cdot \mathbf{i} + y(t) \cdot \mathbf{j} + z(t) \cdot \mathbf{k} \quad (2) \]

\[ \mathbf{r}(t) = \begin{bmatrix} a_x \cdot (t-t_s)^0 + s_x \cdot (t-t_s)^1 - (t-t_s)^1 \\
a_y \cdot (t-t_s)^0 + s_y \cdot (t-t_s)^1 - (t-t_s)^1 \\
a_z \cdot (t-t_s)^0 + s_z \cdot (t-t_s)^1 - (t-t_s)^1 \end{bmatrix} \quad (3) \]

Note that Equation 3 is essentially four dimensional (4D), because it consists of three spatial dimensions plus one time dimension. When plotting a 4D function in a 3D spatial space, the time dimension will be omitted (Figure 3a). Analogously, if construction activities are conducted on a 2D plane, the third spatial dimension can be omitted (Figure 3b). In this case, a point-vector form of singularity functions is applied. The start point coordinates of a work path are \((a_x, a_y, t_s), \) where \( a_x \) and \( a_y \) are spatial values and \( t_s \) is the start time. The finish point coordinates of the same work path are \((a_x, a_y, t_F). \) Duration \( D \) of the activity that moves along the work path is the
difference between $t_F$ and $t_S$. Thus, the growth vector $(s_x, s_y, s_z)$ is 

$$((a_x - a_x')/D, (a_y - a_y')/D, 1).$$

The point-vector form of a work path in the space-space-time (SST) space thus is given by Equation 4. This paper focuses on activities that occur on 2D plane.

But the new model could also be used to plan activities that are carried out in a truly 3D space.

$$Path(t)_{SST} = \left[ \begin{array}{l}
    a_x \cdot (t-t_s)^0 + \frac{a_x' - a_x}{D} \cdot (t-t_s)^1 - (t-t_f)^1, \\
    a_y \cdot (t-t_s)^0 + \frac{a_y' - a_y}{D} \cdot (t-t_s)^1 - (t-t_f)^1, \\
    t_S \cdot (t-t_s)^0 + 1 \cdot (t-a_{s})^1 - (t-a_{s'})^1 \\
\end{array} \right]$$ (4)

**Definition of Work Areas**

Based on several considerations, singularity functions will be used here to define the work areas:

- A geometric shape can be partially switched on or off on any axis in a time-space chart, which allows monitoring and controlling ongoing partially completed activities (Figure 4d).

- As will explained in detail, signal functions can be defined for each geometric shape, which in turn allows the definition of Boolean operations between such different geometric shapes.

- Computational geometry employs dissimilar algorithms, e.g. polygon triangulation. The new mathematical model, on the other hand, provides a unified way to define and manipulate many geometric shapes for spatial-temporal modeling in the construction planning domain.

The process for defining the three-dimensional geometric shape representing a work area is described in the flowchart of Figure 5. The known variables are the start point $(a_x, a_y, a_z)$ and three offset functions $Offset_1$, $Offset_2$, and $Offset_3$. Offset functions define a displacement direction and amount for specific purposes: $Offset_1$ connects a start and finish point, which gives a line segment (Figure 4a); $Offset_2$ connects such a line segment with another line segment, which gives a plane (Figure 4b); $Offset_3$ connects it to another plane, which gives a prism (Figure
4c). Equation 5 is the offset function $Offset_1$. In this function a vector $(\Delta_x, \Delta_y, \Delta_z)$ represents the change in three dimensions. $Offset_2$ and $Offset_3$ are analogous to $Offset_1$, but with indices 2 and 3.

\[
Offset_1 = h_S \cdot \langle h_1 - h_{iS} \rangle^R + \left( \begin{array}{c}
\Delta_{1x} \\
\Delta_{1y} \\
\Delta_{1z}
\end{array} \right) \cdot \left( \langle h_1 - h_{iS} \rangle^R - \langle h_1 - h_{iF} \rangle^R - \langle h_1 - h_{iF} \rangle^L \right)
\]

\[
= h_S \cdot \langle h_1 - h_{iS} \rangle^R + \left( \begin{array}{c}
\Delta_{1x} \\
\Delta_{1y} \\
\Delta_{1z}
\end{array} \right) \cdot \left( \langle h_1 - h_{iS} \rangle^R - \langle h_1 - h_{iF} \rangle^R - \langle h_1 - h_{iF} \rangle^L \right)
\]

\[
= h_S \cdot \langle h_1 - h_{iS} \rangle^R + \Delta_{1x} \cdot \left( \langle h_1 - h_{iS} \rangle^R - \langle h_1 - h_{iF} \rangle^R - \langle h_1 - h_{iF} \rangle^L \right) + \Delta_{1y} \cdot \left( \langle h_1 - h_{iS} \rangle^R - \langle h_1 - h_{iF} \rangle^R - \langle h_1 - h_{iF} \rangle^L \right)
\]

\[
+ \Delta_{1z} \cdot \left( \langle h_1 - h_{iS} \rangle^R - \langle h_1 - h_{iF} \rangle^R - \langle h_1 - h_{iF} \rangle^L \right)
\]

The independent variables $h_1$ in Equation 5 is a dummy variable, whose domain is from $h_S$ to $h_F$. Normally, $h_S$ and $h_F$ are 0 and 1 respectively, but they can have any value within [0, 1] and of course $h_S > h_F$. This allows modeling a fractional part of a shape (a truncated prism), which is necessary to monitor and control the process while activities have been partially completed (Figure 4d). Conceptually the offset, line, plane, and prism functions can be discretized per Figure 5, so that it could be stored in 3D matrices for computerization. Each cell in a matrix could contain point coordinates for a geometric shape. Yet in this research a continuous model of offset, line, plane, and prism as singularity functions is more advantageous. Adding the first offset function to the start point gives a line segment per Equation 6. Adding the second gives a plane per Equation 7. And the third gives a prism per Equation 8. This fulfills Objective 2.

\[
Work \ path = Line = Start \ point + Offset_1
\]

\[
Work \ path \ with \ safety \ buffer = Plane = Line + Offset_2
\]

\[
Work \ area = Solid = Plane + Offset_3
\]

\[
= Line + Offset_2 + Offset_3 = Point + Offset_1 + Offset_2 + Offset_3
\]

**Definition of Temporal and Spatial Relationships**
Execution needs temporal and spatial relationships among overlapping and dependent activities. Temporal relationships are represented as vertical distances in the time-space chart, and spatial relationships as horizontal distances. By mathematically modeling these spatial constructs with singularity functions, such distances can be precisely planned, measured, and controlled. As the literature review noted, the network model of CPM and other scheduling methods only contain temporal relationships between the activity endpoints, i.e. their starts and finishes. But such relationships cannot express the concurrent activities that are widespread in construction projects. The present research addresses this issue by defining temporal point-to-point relationships that can connect internal points of overlapping activities (Hajdu et al. 2017, Hajdu 2015, Francis and Miresco 2002). Such points represent both moments in time and locations on site. The units into which the activity is divided can differ in size, and their size is represented in the model by such mid-activity points. For example, if a crew can enter a work area for its repetitive activity as soon as another crew finishes there, their activities are linked with a temporal relationship. Naturally, traditional FS, SS, SF and FF links can also be defined. In the new model, temporal (vertical) relationships between two work paths or work areas are defined as follows: \( \text{Work path}_2 = \text{Work path}_1 + \text{Time buffer}; \text{Work area}_2 = \text{Work area}_1 + \text{Time buffer} \). Spatial relationships have so far been mostly defined as SS and FF relationships in location-based schedules. Since these schedules are based on a particular sequence of work areas or on a single work path, such relationships can be interchanged with temporal ones. A distance between two crews who move along the same path, for example, can also be represented as an equivalent time lag. However, other kinds of spatial relationships are useful and must be explicitly defined, because they restrict the solution space. For example, a paving crew may be required to always maintain a minimum safe distance from earthmoving equipment that works along another path.
Such spatial relationships are defined as horizontal constraints in the time-space chart, e.g. as the minimum required horizontal distance between two work paths or work areas. To illustrate how such temporal and spatial relationships are defined, Appendix A provides two detailed examples.

**Distance Calculation**

Assume work paths A and B per Equation 9. Path B is analogous to Path A, but with index B. Horizontal and vertical (space and time) distances along these paths can be calculated as follows (Isaac et al. 2017): First, Equation 10 finds their Euclidian distance, where $A_x, A_y, A_t, B_x, B_y, B_t$ are components in these two vector equations. Second, Equation 11 calculates first-order and second-order partial derivatives of $h_A$ and $h_B$ to identify where the closest distance ($P_{AB}$) occurs. Third, the constraint is applied that the vertical components of the paths are set equal ($A_t = B_t$), which returns the horizontal distance. A temporal distance only makes sense if both paths pass the same spatial point (projections cross in the $x$-$y$ plane). Deleting time in Equation 9 gives $x$-$y$-projections in Equation 12. Setting them equal ($A_{xy} = B_{xy}$) gives any crossing ($x^*, y^*$) if it exists.

Inputting it into path equations gives times $t_A$ and $t_B$, whose difference is the temporal distance.

$$A = \left(\begin{array}{c} a_{A_x} \\ a_{A_y} \\ a_{A_t} \end{array}\right) + \left(\begin{array}{c} \Delta_{1xA} \\ \Delta_{1yA} \\ \Delta_{1tA} \end{array}\right) \cdot \left(\begin{array}{c} (h_A - 0)^1_R - (h_A - 1)^1_R - (h_A - 1)^0_L \\ \end{array}\right)$$

(9)

$$P_{AB}^2 = (B_x - A_x)^2 + (B_y - A_y)^2 + (B_t - A_t)^2$$

(10)

$$\frac{\partial}{\partial h_A} P_{AB}^2 (h_A, h_B) = 0$$

if $$\frac{\partial}{\partial h_A} \left( \frac{\partial}{\partial h_A} P_{AB}^2 (h_A, h_B) \right) > 0$$ and $$\frac{\partial}{\partial h_B} \left( \frac{\partial}{\partial h_A} P_{AB}^2 (h_A, h_B) \right) > 0$$

(11)

where the curve $P_{AB}$ that Equation 10 traces reaches its minimum value. This fulfills Objective 3.

$$A_{xy} = \left(\begin{array}{c} a_{A_x} \\ a_{A_y} \end{array}\right) + \left(\begin{array}{c} \Delta_{1xA} \\ \Delta_{1yA} \end{array}\right) \cdot \left(\begin{array}{c} (h_A - 0)^1_R - (h_A - 1)^1_R - (h_A - 1)^0_L \\ \end{array}\right)$$

(12)

**Relationships Based on Logical Operators**
The new model provides definitions for versatile relationships between activities with its logical operators. Besides the aforementioned material implication, the following operators can be used:

- If two activities must be executed jointly, then use a logical conjunction (AND);
- If two activities may (but need not) be executed jointly, then use a logical disjunction (OR);
- If two activities must not be jointly executed, then use an exclusive disjunction (XOR).

Such Boolean logical operations will be defined using signal functions. Singularity terms that serve as signals have been used successfully to model periodical phenomena in cash flows, e.g. interest and payments (Su and Lucko 2015). The basic concept of signal functions is to define a ‘window’ at the required times or points in space, where the signal is switched on (i.e. value = 1) and off (i.e. value = 0). Using signal functions to create Boolean operators has two advantages:

1. The signal value is either 0 or 1, which reflects the traditional inputs of Boolean operations;
2. Signal functions can be multiplied with the geometric functions to give a customized result.

Signal functions for three dimensions are defined in Equation 13, which has two terms. \( \text{Signal}(y) \) and \( \text{Signal}(z) \) are analogous to \( \text{Signal}(x) \), but with indices \( y \) and \( z \). The first and second cutoffs are a start and a finish. Their difference is a range; if independent variables \( x, y, \) or \( z \) are within it, the signal is 1, else 0. This signal can be multiplied with the spatial constructs of Equations 6-8.

\[
\text{Signal}(x) = \langle x - a_{ss} \rangle^0 - \langle x - a_{sf} \rangle^0
\]  

(13)

The definitions of Boolean operators and the corresponding signal functions are listed in Table 3. Figure 6 shows examples for relationships between two different work areas (for different activities). In each case, the area where activities can be performed is shown for a given relation.

Once the activities and their relationships have been defined using the constructs described
above, the model can be used to identify violations of those relationships (Step 3 in the process described in Figure 1) and to improve the efficiency of the plan (Step 4). It is important to note that the temporal-spatial analysis in Steps 3 and 4 should alternate recursively, because a time reduction (or increase) might violate spatial constraints, and *vice versa*. Solutions already exist for automatically solving such a compacting problem (e.g. Fasano 2004), and these could be incorporated into the model in future research to ensure that an optimal solution is reached in a single step. This research will conduct an iterative ‘What-If’ analysis of temporal or spatial constraints, precedence, and Boolean relationships between activities. As the next section will illustrate, 3D plots can support a ‘What-If’ analysis by displaying temporal and spatial criticality. The mathematical formulation also allows customized visualizations. This fulfills Objective 4.

**Implementation and Validation**

A real-world case study of a high-rise building was chosen for the validation of the model. The mathematical model that has been described was implemented in MATLAB code. The project network model is automatically defined with BIM data per *Definition of Activity Attributes Using BIM Data*. This model includes the spatial constructs and the Boolean relationships that the formal schedule in the case study lacked, but that still informally guided the actual execution.

**Manual Approach Used in Actual Case Study**

This project for validation is chosen both for its size and being representative of many projects around the world, particularly in Asia and the Middle East (Allianz 2014), which makes it ideal to validate the new model. Since the spatial aspects cannot be represented in the current network model, CPM and Gantt charts are insufficient tools to aid in the actual project execution. While of course the project had a formal CPM schedule, per the authors’ observations it was not used for planning the execution of the case study project. Instead, it was relegated to being a mere
progress recording tool. In other words, it was kept updated at certain intervals for potential negotiation or litigation vis-à-vis the owner and subcontractors. Without the foundation of a comprehensive model, scheduling therefore resorted to ad-hoc analysis and informal decision-making. In the case study project the team improvised such a manual tool as follows: they hung a large poster matrix on the wall of their office, to which they attached colored paper sticker notes to indicate the stage of execution in each activity and location. While it was quick and cheap, held an intuitive appeal and fulfilled its purpose, such improvised approach has many drawbacks:

- It cannot be generalized for different projects, but has to be custom-created for a desired task;
- It cannot be shared across different offices, because it only exists as a single physical edifice;
- It cannot automatically create a solution that fulfills all applicable time and space constraints;
- It cannot be used with any optimization algorithm, because it is not in a mathematical format;
- It cannot be integrated with portable digital devices (which were used extensively for control purposes) to identify productivity trends, nor predict how they impact the finish date;
- It cannot generate or compare multiple options or scenarios for performing What-If analyses;
- It cannot be used as input for computerized calculations unless all of its data are transcribed;
- It cannot provide simulation capabilities to analyze realistic probabilistic activity durations.

By creating an integrated project planning model based on a rigorous mathematical formulation, this research will support implementing such analyses, which current manual methods preclude.

**Model Implementation**

A real-world case study of a residential building project is used to implement and test the new model (details of the case study are described Appendix B). Figure 7 shows a partial network for a single floor of the building. Note that only the nodes colored in gray are activities that are included in this case study. Figure 8 shows a 3D view of the time-space chart for the
resultant profiles of the activities that were modeled. Activities that are critical in temporal and
spatial terms are framed in red in Figure 8 (e.g. Aluminum Window installation in the left and
right apartments on the top floor). Unlike time juxtaposition plots (Bach et al. 2014), these charts
allow the planner to fully identify which activities are active where and when. Note especially
that the vertical axis is time, not height of the building. The different shapes of the activities can
be explained from left to right and top to bottom as follows. Lower floors are worked on earlier,
so that they appear lower on the time axis. Other floor sequences would be feasible. Activities
that occur on the façade such as sealing appear like an envelope over the building footprint.

<INSERT FIGURE 7 HERE>

<INSERT FIGURE 8 HERE>

The central combined 3D chart shows a vertical diagonal section of the time-space chart (half
invisible so that its interior can be seen). As the model shall plan as well as control, incorporating
changes that occur during project execution must be tested. It updates the time-space-criticality
evaluation, searching for new violations of temporal and spatial relationships after any change.
One such change involved a subcontractor who didn’t follow the project manager’s strategy of
maintaining a constant rate of one floor weekly for the activities, and who instead waited for the
predecessor activity to be completed for a batch of floors, and then carried out his activities in
one continuous flow (see Appendix B). The model can compare the impact of this change in
actual execution. The results of implementing the model showed that criticality had indeed
changed, with some activities being newly critical, whereas others were no longer critical.

To test the ability to identify violations, an artificial change can be inserted, e.g. a delayed
start of HVAC installation in the left apartment on Floor 36 from days 7 to 10. A violation of
XOR between HVAC and Plumbing installation is correctly found on all floors (red in Figure 9).
A comparison is also made with the results that could be obtained from existing planning with CPM. A CPM analysis is carried out with a conventional activity network model of the planned activities that included only temporal precedence relationships (Figure 10). This analysis results in a total duration of 47 days. In contrast, the new model, in which spatial constructs and Boolean relationships had been incorporated, plans the execution of the same activities within 41 days. When an updated network is analyzed with CPM, incorporating changes in the actual execution, the result is 68 days, as opposed to 64 days with the new model.

The reason for this difference in the results obtained with the conventional model and the proposed model is that the first allowed only the definition of strict sequential relations between activities, whereas the proposed approach uses Boolean operators in a more realistic way to define relations in both temporal and spatial dimensions. Using CPM to prepare a schedule, implicit safety considerations are implied indirectly through temporal relationships. Its limitations prevent, for example, two activities from occurring simultaneously on the same floor. Yet with the proposed model an OR Boolean operator can be used to allow them to be conducted concurrently in both temporal and spatial dimensions without creating any new safety hazards.

The proposed model thus supports an analysis of the time-space-criticality of activities, and not just of the time-criticality as CPM does. As a result, different activities may be identified as being critical. For example, the Aluminum Window and Stone Cladding activities on the 5th floor were identified as being temporally and spatially critical with the new model, whereas with CPM they were incorrectly identified as being non-critical (the activities on the facade are indicated with a red frame in Figure 10, and the critical activities are emphasized with a thick frame). Using CPM thus misleads the planner, resulting in an unrealistic and less efficient plan.
Contributions to Body of Knowledge and Conclusions

The proposed model makes several contributions to the body of knowledge. The integrated planning model incorporates important information that cannot be defined in any single existing one. Complex spatial information on the location and movement of activities on the site over time is captured by defining work paths and work areas, as well as the buffers between them. The model also extends the understanding of relationships that go beyond traditional precedence, namely spatial and temporal point-to-point relationships and Boolean logical operators, instead of merely the material implication. An extraction process from BIM has been demonstrated.

This proposed approach is essential to support effective execution planning in construction projects. The model can be used to create a schedule that takes into consideration important factors that cannot currently be represented and therefore are unfortunately ignored in CPM calculations. This confirms the research hypothesis. Moreover, the model has potential to be used for other types of analysis, such as e.g. site safety or resource allocation. As a mathematical formulation it is extensible to further aspects and applications. And it is customizable to generate any useful visualization of the temporal-spatial data in the new project planning model.

Recommendations for Future Research

Advanced sensing technologies are being developed to capture the actual progress in construction projects and automate monitor processes (e.g. Brilakis et al. 2011, Golparvar-Fard et al. 2009, Turkan et al. 2012). Future research should explore the use of the model for such control purposes. This research has not provided explicit approaches, i.e. procedures for plan updating in light of deviations that occur during execution, identifying such performance trends, and derive appropriate control actions, all of which should be the subject of future research.
Another item is to explore conceptual similarities and differences of all three possible types – time-criticality, time-space-criticality, and space-criticality, which this research has only briefly and partially touched. Their counterparts in terms of float must of course also be delineated and may give measures of resistance to delays as well as efficiency of using time and space. An optimization algorithm could generate a schedule with a given quantity of float, depending on the planner’s level of risk averseness. In conjunction with this the computational efficiency of different possible approaches for stacking and compacting activities in such 3D manner should be investigated.

Moreover, future research should also address how the model could handle aspects that are traditionally addressed in site layout planning, but which clearly affect the detailed execution of projects. These aspects include the location and duration of major temporary resources on site, e.g. cranes, material stockpiles, and scaffolding. Furthermore, the financial dimension of cost has been beyond the scope of this research and should be added to gain a powerful integrated tool for project management. Finally, while the new model has extended feasible constraints, they are only minimum constraints, so that maximum constraints could still be added (Hajdu 2015).

Data Availability Statement

Data generated or analyzed during the study are available from the corresponding author by request.

References


Table 1: Limitations of Current Scheduling Methods

<table>
<thead>
<tr>
<th>Domain</th>
<th>Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relationships between activities</td>
<td>Link only activity endpoints and ignore internal points</td>
</tr>
<tr>
<td>Logical operators</td>
<td>Limited to material implication (IF-THEN)</td>
</tr>
<tr>
<td>Location-based scheduling</td>
<td>Limited to a single path or repetitive unit on site</td>
</tr>
<tr>
<td>4D Scheduling</td>
<td>Does not fully integrate temporal and spatial information</td>
</tr>
<tr>
<td>Work-space planning</td>
<td>Does not define work areas at activity level</td>
</tr>
</tbody>
</table>
Table 2: Comparison with Previous Study

<table>
<thead>
<tr>
<th>Isaac et al. (2017)</th>
<th>Advances in Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Introduced use of 3D singularity functions to model work paths</td>
<td>• Uses 4D singularity functions to model work areas and work paths</td>
</tr>
<tr>
<td>• Represented paths in time-space chart</td>
<td>• Includes spatial relationships in 3D space, not only on single horizontal plan</td>
</tr>
<tr>
<td>• Calculated vertical temporal and horizontal spatial distances were between work paths</td>
<td>• Uses Boolean operators to either allow or prevent certain overlaps between activities</td>
</tr>
<tr>
<td>• Used manual input</td>
<td>• Incorporates BIM data as model input</td>
</tr>
</tbody>
</table>
### Table 3: Boolean Operations and Signal Functions

<table>
<thead>
<tr>
<th>Operator</th>
<th>Type</th>
<th>Notation</th>
<th>Rule</th>
<th>Signal Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material implication</td>
<td>IF-THEN</td>
<td>$x \rightarrow y$</td>
<td>If $x = 1$ and $y = 0$, then $x \rightarrow y = 0$, else 1</td>
<td>N/A</td>
</tr>
<tr>
<td>Conjunction</td>
<td>AND</td>
<td>$x \land y$</td>
<td>If $x = y = 1$, then $x \land y = 1$, else 0</td>
<td>Signal_A \times Signal_B</td>
</tr>
<tr>
<td>Disjunction</td>
<td>OR</td>
<td>$x \lor y$</td>
<td>If $x = y = 0$, then $x \lor y = 0$, else 1</td>
<td>Signal_A + Signal_B - (Signal_A \times Signal_B)</td>
</tr>
<tr>
<td>Exclusive disjunction</td>
<td>XOR</td>
<td>$x \oplus y$</td>
<td>If $x = y$, then $x \oplus y = 0$, else 1</td>
<td>OR signal - AND signal</td>
</tr>
<tr>
<td>Case</td>
<td>Start Point</td>
<td>$\Delta_1$</td>
<td>$\Delta_2$</td>
<td>$\Delta_3$</td>
</tr>
<tr>
<td>----------------------</td>
<td>-------------</td>
<td>-------------------------------</td>
<td>-------------</td>
<td>------------</td>
</tr>
<tr>
<td>Work paths</td>
<td>(1, 1, 0)</td>
<td>1st segment (3, 1, 1)</td>
<td>(0, 0, 0)</td>
<td>(0, 0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd segment (4, 3, 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Work path with safety buffer</td>
<td>(1, 1, 0)</td>
<td>1st segment (3, 1, 1)</td>
<td>(0, 3, 0)</td>
<td>(0, 0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd segment (4, 3, 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Work area</td>
<td>(1, 1, 0)</td>
<td>1st segment (3, 1, 1)</td>
<td>(0, 3, 0)</td>
<td>(0, 0, 0.5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd segment (4, 3, 2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table A2: Inputs of Figure A2 (Spatial Distance = 3 length units)

<table>
<thead>
<tr>
<th>Case</th>
<th>Start Point</th>
<th>$\Delta_1$</th>
<th>$\Delta_2$</th>
<th>$\Delta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work paths</td>
<td>(4, 0, 0)</td>
<td>1\textsuperscript{st} segment (4, 3, 1) 2\textsuperscript{nd} segment (2, 2, 2)</td>
<td>(0, 0, 0)</td>
<td>(0, 0, 0)</td>
</tr>
<tr>
<td>Work path with safety buffer</td>
<td>(4, 0, 0)</td>
<td>1\textsuperscript{st} segment (4, 3, 1) 2\textsuperscript{nd} segment (2, 2, 2)</td>
<td>(-0.6, 0.8, 0)</td>
<td>(0, 0, 0)</td>
</tr>
<tr>
<td>Work area</td>
<td>(4, 0, 0)</td>
<td>1\textsuperscript{st} segment (4, 3, 1) 2\textsuperscript{nd} segment (2, 2, 2)</td>
<td>(-0.6, 0.8, 0)</td>
<td>(0, 1)</td>
</tr>
</tbody>
</table>
### Table B1: Relationships between Activities in Each Floor

<table>
<thead>
<tr>
<th>Name</th>
<th>Color</th>
<th>Planned Duration</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural Work</td>
<td>Green</td>
<td>6 d/floor</td>
<td>Blind Frame: FS – 0</td>
</tr>
<tr>
<td>Blind Frame</td>
<td>Blue</td>
<td>1 d/floor</td>
<td>Sealing</td>
</tr>
<tr>
<td>Sealing</td>
<td>Blue</td>
<td>6 d/floor</td>
<td>Aluminum Window Left: FS – 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sealing (next floor): FS – 0</td>
</tr>
<tr>
<td>HVAC Left Apartment</td>
<td>Yellow</td>
<td>3 d/floor</td>
<td>Plumbing Left: <strong>XOR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aluminum Window Left: <strong>OR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HVAC Right: FS – 0</td>
</tr>
<tr>
<td>Aluminum Window Left</td>
<td>Cyan</td>
<td>1 d/floor</td>
<td>Plumbing Left: <strong>OR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aluminum Window Right: FS – 0</td>
</tr>
<tr>
<td>Plumbing Left</td>
<td>Magenta</td>
<td>3 d/floor</td>
<td>Plumbing Right: FS – 0</td>
</tr>
<tr>
<td>HVAC Right Apartment</td>
<td>Yellow</td>
<td>3 d/floor</td>
<td>Plumbing Right: <strong>XOR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aluminum Window Right: <strong>OR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HVAC Left (Next Floor): FS – 0</td>
</tr>
<tr>
<td>Aluminum Window Right</td>
<td>Cyan</td>
<td>1 d/floor</td>
<td>Plumbing Right: <strong>OR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aluminum Profile: FS – 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aluminum Window Left (Next Floor): FS – 0</td>
</tr>
<tr>
<td>Plumbing Right</td>
<td>Magenta</td>
<td>3 d/floor</td>
<td>Plumbing Left (Next Floor): FS – 0</td>
</tr>
<tr>
<td>Aluminum Profile</td>
<td>Cyan</td>
<td>1 d/floor</td>
<td>Stone cladding: <strong>XOR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aluminum Profile (Next Floor): FS – 0</td>
</tr>
<tr>
<td>Stone Cladding</td>
<td>Green</td>
<td>1 d/floor</td>
<td>Stone Cladding (Next Floor): FS – 0</td>
</tr>
</tbody>
</table>
1. Define Activity Attributes Using BIM Data:
   - Duration
   - Temporal constraints
   - Spatial coordinates and requirements
   **Work areas in BIM**

2. Define Relationships:
   - Precedence relationships (FS, SS, FF, SF) with time buffer (lead/lag) if required
   - Boolean operators (AND/OR/XOR)
   **Work areas with safety buffer**

3. Detect Violations:
   Resolve by adding time or space if necessary

4. Improve Efficiency:
   Reduce distances in time or space if possible

**Figure 1: Temporal-Spatial Planning Process**
Figure 2: Example of Work Areas Defined in BIM
Figure 3: Plots of Work Paths Defined with Singularity Functions

(a) Work path with three spatial dimensions

(b) Work path with two spatial and time dimensions
Figure 4: Line, Plane, Prism, and Truncated Prism

(a) Line (representing a work path)

(b) Plane (work path with safety buffer)

(c) Prism (work area)

(d) Prism truncated at 50% (work area with partially completed activity)
1: Known variables
Start point \((a_xS, a_yS, a_zS)\) and
\(Offset_1, Offset_2\) and \(Offset_3\)

2: First offset generates line
Start point \((a_xS, a_yS, a_zS)\)
Finish point = start point + \(Offset_1\)

3: Second offset generates plane
Start line (shape from step 2)
Finish line = start line + \(Offset_2\)

4: Third offset generates prism
Start plane (shape from step 3)
Finish plane = start plane + \(Offset_3\)

Figure 5: Flowchart to Define Prism
Figure 6: Logical Operations Between Two Work Areas

(a) Two work areas

(b) Area 1 ∧ Area 2 (AND)

(c) Area 1 ∨ Area 2 (OR)

(d) Area 1 ⊕ Area 2 (XOR)
Figure 7: Partial Activity Network for One Floor
Figure 8: Temporal and Spatial Model (Original Plan)
Figure 9: Violation Check
Figure 10: Planned Conventional Network
Figure A1: Examples of Temporal Relationships

(a) Work paths  
(b) Work paths with safety buffer  
(c) Work areas

Temporal constraint of work area

Time buffer
Figure A2. Examples of Spatial Relationships
Figure B1: Work Areas on Each Floor
Figure and Table Captions

Figure 1: Temporal-Spatial Planning Process
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   (a) Two work areas
   (b) Area 1 \( \land \) Area 2 (AND)
   (c) Area 1 \( \lor \) Area 2 (OR)
   (d) Area 1 \( \oplus \) Area 2 (XOR)
Figure 7: Partial Activity Network for One Floor
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Figure A1: Examples of Temporal Relationships
   (a) Work paths
   (b) Work paths with safety buffer
   (c) Work areas
Figure A2. Examples of Spatial Relationships
   (a) Work paths
   (b) Work paths with safety buffer
   (c) Work areas
Figure B1: Work Areas on Each Floor

Table 1: Limitations of Current Scheduling Methods
Table 2: Comparison with Previous Study
Table 3: Boolean Operations and Signal Functions
Table A1: Inputs of Figure A1 (Time Buffer = 3 days)
Table A2: Inputs of Figure A2 (Spatial Distance = 3 length units)
Table B1: Relationships between Activities in Each Floor
Appendix A: Examples of Temporal and Spatial Relationships

The relevant calculations for the two work paths defined in Table A1 are shown in Equations A1-A2, and the time-space charts in Figure A1. Note that the time buffer is 3 days and each path and area consists of two segments with a bend in the middle.

\[
{\text{Work path}}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \cdot \left( \langle h_1 - 0 \rangle^1_R - \langle h_1 - 1 \rangle^1_R - \langle h_1 - 1 \rangle^0_L \right) + \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \cdot \left( \langle h_1 - 1 \rangle^1_R - \langle h_1 - 2 \rangle^1_R - \langle h_1 - 2 \rangle^0_L \right) \tag{A1}
\]

\[
{\text{Work path}}_2 = \text{Work path}_1 + \text{Time buffer} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \cdot \left( \langle h_1 - 0 \rangle^1_R - \langle h_1 - 1 \rangle^1_R - \langle h_1 - 1 \rangle^0_L \right) + \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} \cdot \left( \langle h_1 - 1 \rangle^1_R - \langle h_1 - 2 \rangle^1_R - \langle h_1 - 2 \rangle^0_L \right) \tag{A2}
\]

The work path with a safety buffer is defined as by adding Offset to the work path defined in Equation A1, where \(\text{Offset} = (0, 3, 0) \cdot \left( \langle h_2 - 0 \rangle^1_R - \langle h_2 - 1 \rangle^1_R - \langle h_2 - 1 \rangle^0_L \right)\) with the actual input values of this example. The second work path with safety buffer is then defined by adding a time buffer to the first work path with safety buffer. The work area is defined in turn by adding Offset to the work path of Equation A1, where \(\text{Offset} = (0, 0, 0.5) \cdot \left( \langle h_3 - 0 \rangle^1_R - \langle h_3 - 1 \rangle^1_R - \langle h_3 - 1 \rangle^0_L \right)\).

An example of the definition of spatial relationships is provided as well, based on the inputs in Table A2, and the relevant time-space charts in Figure A2. The work path is defined as:

\[
{\text{Work path}}_1 = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \cdot \left( \langle h_1 - 0 \rangle^1_R - \langle h_1 - 1 \rangle^1_R - \langle h_1 - 1 \rangle^0_L \right) + \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \cdot \left( \langle h_1 - 1 \rangle^1_R - \langle h_1 - 2 \rangle^1_R - \langle h_1 - 2 \rangle^0_L \right) \tag{A3}
\]
The minimum required horizontal distance between the two work paths is defined by adding a spatial vector to Work path_1, where \( \text{Spatial vector} = (x, y, 0) \), \( x^2 + y^2 = 3^2 \) and \( (x, y, 0) \).

\[
(4, 3, 1) = 0 \quad \text{to give} \quad x / y = -3/4, \quad \text{so that} \quad \text{Spatial vector} = (-1.8, 2.4, 0) \quad \text{or} \quad (1.8, -2.4, 0).
\]

The work path with safety buffer is defined by adding \( \text{Offset}_2 \) to Equation A3, where \( \text{Offset}_2 = (-0.6, 0.8, 0) \) \cdot \left( \langle h_z - 0 \rangle^1_r - \langle h_z - 1 \rangle^1_r - \langle h_z - 1 \rangle^0_L \right) \), \( \Delta_2 = (x, y, 0) \), and \( x^2 + y^2 = 1 \) and \( (x, y, 0) \).

\[
(4, 3, 1) = 0, \quad \text{to give} \quad x / y = -3/4, \quad \text{so that} \quad \Delta_2 = (-0.6, 0.8, 0) \quad \text{or} \quad (0.6, -0.8, 0).
\]

For Work path_2 with safety buffer, add the spatial vector to Work path_1. For Work area_1, add \( \text{Offset}_2 + \text{Offset}_3 \) to Equation A3, where \( \text{Offset}_2 = (-0.6, 0.8, 0) \) \cdot \left( \langle h_z - 0 \rangle^1_r - \langle h_z - 1 \rangle^1_r - \langle h_z - 1 \rangle^0_L \right) \) \quad \text{and} \quad \text{Offset}_3 = (0, 0, 1).

\[
\left( \langle h_z - 0 \rangle^1_r - \langle h_z - 1 \rangle^1_r - \langle h_z - 1 \rangle^0_L \right). \quad \text{Finally, for Work area}_2, \quad \text{add the spatial vector to the Work area}_1.
\]

Appendix B: Description of Case Study

In this residential project a 156-meter-tall, 45 floor building was constructed, which included 164 apartments (the location of the project is not disclosed for reasons of anonymity). It has a total area of 30,000 m\(^2\) and is located on top of four subterranean parking floors with 360 parking spaces on 14,000 m\(^2\). A typical floor is 855 m\(^2\), of which the core with elevators, stairs, and service shafts covers 170 m\(^2\). Milestones were:

- Completion of building structure after 36 months;
- Completion of finishes after 41 months (mostly parallel to structural activities);
- Final project delivery to owner after 49 months.
The structural system of the building is cast-in-place reinforced concrete. Its envelope combines stone cladding and aluminum glazing. Among the many activities this case study will focus on four interior finishing activities and three external cladding activities for validation, because they implicitly exhibit a Boolean logic, even though the original schedule could not represent it.

The building has a double tall ground floor with lobby and pool, six apartments on each of floors 3-16, four on each of floors 17-35, two on floors 36-40 (selected for analysis herein), two triplex penthouse apartments that span from floor 41 to 43, and two roof levels with a setback for mechanical equipment. For a concise graphical presentation, the following section treats only the five selected floors. Each contains two apartments. Each apartment is treated as a separate work area (Figure B1). Besides the floors that this paper analyzes, other work areas and indeed footprints of buildings in other regions can be modeled in analogy to the following description.

Three activities are performed in each of the apartments, each by a different subcontractor: HVAC duct installation; plumbing pipe installation; and aluminum window installation. The first two activities are preceded by the installation of piping and ducting along vertical work paths in service shafts within the building core. These pipes and ducts serve the apartments on each floor. To avoid physical inference, HVAC duct and plumbing pipe installation cannot be concurrent in the same work area, but either one could occur first (i.e. an XOR relation). Yet window installation can occur simultaneously to any of the other two activities in that work area (i.e. OR relation). It is preceded by the installation of blind frames.

Two other activities move on external vertical work paths, with crews on two mast-climbing work platforms on each side of the façade:

- Window sealing (after installing the window blind frames from inside the apartments);
• Aluminum profile installation (after the installation of windows from inside the apartments). These activities are carried out sequentially. Aluminum Window installation moves from one work area to another, while the related Sealing of the windows moves on a vertical work path on the exterior of the building. Another activity, Stone Cladding, also uses the work platforms, but therefore cannot be carried out simultaneously with the two other external activities (i.e. a XOR relationship) on the same section of a façade. Temporal and Boolean relationships between selected activities on a typical floor are listed in Table B1. Unlike CPM, Boolean relationships can consider both temporal and spatial constraints.

<INSERT TABLE B1 HERE>

In the case study, the project manager adopted a strategy of maintaining a constant rate of one floor weekly (6 days) for most activities. The rate was driven by the speed of the Structural Work, which was meant to simplify the management of interfaces with subsequent finishing activities, despite the fact that it could cause idle time for these subcontractors. Thus, one week was allocated to seal each floor. In practice, the enclosure subcontractor who installed frames, sealant, and windows waited until the structural subcontractor had completed a batch of floors. In each batch of floors the enclosure subcontractor installed frames, sealant, and window in one continuous flow. Thus Sealing now progressed at a rate of three days per floor (but frames and windows retained their rate of Table B1), because the subcontractor was no longer limited by the slower work rate that the project manager had defined. This strategy was more convenient in terms of labor allocation and materials supply, storage, and movement such as frames. However it caused idle time between batches of floors and carried the risk of creating delays in successors.