Reconsidering an Appropriate Probability Distribution Function for Construction Simulation

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ABSTRACT
Simulation in construction management, whether Monte Carlo sampling-based, implemented in the Program Evaluation and Review Technique (PERT) – a probabilistic scheduling method that handles variable activity durations, or used in other applications, relies upon probability density functions (PDF) of little realism like Uniform, simplified like Triangular, idealized like Normal (or Gaussian), or flexible like Beta. But none of these take into account the nature of the system that they model, whose behavior is created by various interactions between project participants in a network-based schedule. This paper therefore examines the recently introduced so-called Tracy-Widom Distribution; derived to mathematically describe the unique behavior of systems that move from mild to intense interactions among their participants, which cause a phase change toward a different behavior. Such characteristics are also found in construction schedules, where a non-critical activity may be briefly delayed without affecting the entire project, or causing to overrun the deadline if critical activities are delayed. The distribution is asymmetrically shaped and steeper on the left side than on the right side. In other words, larger values are more likely than shorter values when sampling. This matches the time extension-prone construction industry and will allow more realistic modeling.

INTRODUCTION
Construction project managers are professionals whose responsibility is to plan, monitor, and control a multitude of technical and non-technical considerations. Basic non-technical dimensions of project management are time and cost. These two dimensions interact and depend on the project-specific scope (Kerzner 2003). Their goal is to complete projects on schedule and within budget. They must follow all specifications to reach the expected quality. Safety is an overarching concern. They use the best available historical data to anticipate and overcome risk and uncertainty.

Construction projects are temporary endeavors created for a specific purpose (PMI 2008), whose inherent complexity and cost intensity necessitate monitoring schedules and budgets. Most projects are deadline-driven, as evidenced by the near ubiquity of ‘time is of the essence’ contract clauses; while conditions of performance remain unpredictable. Uncertainty and circumstances that surround risk compound the time element of the project (Mulholland and Christian 1999). In doing so, the project schedule becomes one of the most important tools for planning and executing.

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Construction Project Schedules

Construction scheduling, as it is currently practiced, originated in the seminal work of Kelley and Walker (1989, 1959), who used one of the first computers to plan an industrial plant. They introduced the Critical Path Method (CPM) that centers on discrete scheduling of time to depict the dependencies of individual tasks into which a project is divided. A CPM schedule is designed to guide participants – the cadre of contractors, subcontractors, etc. – about the relative importance of performing certain activities within the desired completion time (duration) via a predetermined sequence of events. All activities in a network schedule are linked via their starts or finishes.

When activities are linked together to form an overall schedule, variations in activity durations counterbalance each other, thus resulting in a reasonably accurate schedule. Unique to CPM scheduling is activity classification, the determination of predecessor, successor, or concurrent/independent. From this, the critical path, the longest uninterrupted path from beginning to end of the project, is determined; and critical activities are identified: Those along the critical path are those without float or slack time between subsequent activities. Conversely, the remaining activities are deemed non-critical activities: Those not on the critical path with available float.

Construction Risk and Uncertainty

The construction industry is challenged by poor performance due to risk and uncertainty. Projects are increasingly complex, have more schedule demands, employ differing procurement and delivery methods, and face financing limits (Thompson 2012). The construction workplace is a highly dynamic system with numerous interactions between resources and activities over time. Unlike self-contained factories which are static, the construction site changes daily as the project progresses. It is therefore subject to uncertain factors, adding complication (risk) to the process (Campbell et al. 1997). Given the dynamic nature of construction projects, quantification, analysis, and mitigation of risk rise to the forefront of importance. But the reactions thereto remain inconsistent throughout the industry.

Determining what constitutes risk is subjective and influenced by project manager’s past experience and future perception. Factors that shape this assessment weigh heavily on the risk quantification process and impact the path of the project. But they are not well-defined (Tah and Carr 2001). Construction project risk has been classified and categorized in multiple ways. Tah and Carr (2001) present a hierarchy of construction risk as internal versus external, based upon the ability to manage resources versus those relatively uncontrollable. Similarly, Park (1979) had identified twelve specific risks that contractors would normally face, including e.g. the weather, unexpected job conditions, and delays in scheduling and errors in cost estimating.

Risk Manifestation, Management, and Mitigation

From an idyllic project scheduling perspective, the goal of every project is that its activities follow the planned sequence of starts and finishes and are completed on time. This removes the need for time (schedule) adjustments and for flexibility within the sequence of events. However, in the uncertain construction environment, this is not realistic. Risk and uncertainty will typically negatively impact schedule performance, i.e. the completion times of individual activities and the overall project.
CPM schedule provide flexibility by including float – the slack time between the finish of one activity and the start of the next. Consuming float does not impact the ultimate completion of the project (de la Garza et al. 1991). It is a key issue in the completion strategy of a project, is an important means to reduce the impact of risk on projects, and is considered a highly valuable but vanishing commodity (Wickwire et al. 2003). Given this, construction project managers regularly focus on predictive methods and measures for schedule performance. Abramowitz (2009) identified five ways to manage or mitigate risk: Assume it; don’t assume it; abate it; allocate it; or transfer it. While this paper will not prefer one method over another, the commonality among them is that risk must be recognized, quantified, and addressed. It is a method to accomplish this and variations associated therewith on which this paper focuses.

SIMULATION AND MODELING
Simulation in the Construction Industry

Concurrent with the development of CPM, Simon (1957, p. 199) emphasized the limitations of human planning that will inevitably affect all decisions by project managers: “The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behavior in the real world.” Several managerial tools specific to construction project planning arose nevertheless: Bar charts, network-based CPM, and PERT (Aouad and Price 1994). The use of bar charts and CPM is widespread due to their simplicity and ease. Unfortunately, they do not address risk and uncertainty. Simulation, however, has the potential to plan for the unpredictable with more confidence, thus reducing a need for costly buffers (Campbell et al. 1997).

Simulation in construction takes two forms: (1) Simulate repetitive processes / activities as “an estimating tool to assess the productivity of a future operation” (Schexnayder et al. 2005, p. 221), e.g. material preparation (plant operations), moving (hauling), placing (concrete, steel erection, etc.), and cranes (moving, sequence) versus (2) resource planning, i.e. schedule and budget optimization. For the larger project management realm Kim and Reinschmidt (2009) identified two common probabilistic performance forecasts: Cost and schedule. Though simulation is valuable in other fields, e.g. operations-based industries like manufacturing and software development, within construction it has remained within academia (Campbell et al. 1997) at the research stage (Fente et al. 1999), and not widely accepted (Schexnayder et al. 2005).

Campbell et al. (1997, p. 412) found that the construction industry ignored simulation due to “[e]xcessive modelling [sic] time, unrepresentative models and inaccurate results.” They concluded that these were symptomatic of poor simulation management, and that the main reasons for a little simulation in the construction industry were insufficient data, the quality of available simulation software, lacking modeling skills, and unclear methodology, along with the ‘dynamic’ construction site.

A more contemporaneous review of modeling/simulation practice (AbouRizk 2010) found that while notable advances were made since the 1970s, a discrete-event approach prevails. Most success was only academic, with only very minor industry applications beyond limited “‘add-on’ Monte Carlo simulation features” (AbouRizk 2010, p. 1144) to commercial CPM tools. The construction industry lags behind the
operations research state-of-the-art; its simulation could be “an opportunistic tool that should be introduced in opportune applications and situations rather than a universal tool for all construction management applications” (AbouRizk 2010, p. 1152).

Monte Carlo Simulation

Monte Carlo simulation is probably the most common predictive technique to model or simulate possible outcomes in science, engineering, management, finance, and operations. It works by executing a probabilistic system numerous times to obtain statistical inform about its behavior. Such simulations represent uncertainty (risk and/or variation) in parameters of a system under study (herein network construction schedules) as random values that are sampled from estimated probability distribution functions (PDFs). Herein, Monte Carlo simulation will focus on the manifestation of risk and uncertainty in construction projects: Activity delays that create the need to expend float and their impact where none is available – along the critical path. This required analyzing the extent to which project durations are delayed. Most predictable risks occur activity level (Barraza 2011). Schedule simulations can be predictive in nature and identify the extent of schedule risk to be addressed (cf. Abramowitz 2009).

Probability Distribution Functions (PDFs)

The level of predictive success (and the aforementioned accuracy of results) depends on the measures of variability within the simulation model. The reliability of simulation depends on the probability distributions to determine durations (Lee et al. 2013). Schexnayder et al. (2005) concluded that absent of accurate data one must use subjective estimates to model uncertainty. The unpredictable nature of construction leads to subjective selection of PDFs, breeding a lack of confidence in the process (Fente et al. 1999). In turn this limits the practical use of simulation in construction.

Selection Criteria for a PDF

Drawing from parallel research in the operations management arena, Johnson (2002, p. 457) notes that “[p]robably the most basic requirement in risk analysis is to estimate reliably the ‘profile’ of any uncertain quantity in terms of an appropriate cumulative distribution function… and/or its parameters… Most decision-makers are likely to find it difficult to estimate these parameters directly, particularly if the distribution is skewed.” (CDF being the cumulative distribution function for a PDF). At this point, construction schedule and operations research schedule simulation share common ground. Williams (1992) concluded that the question of which distribution to use and which parameters to estimate must be addressed. He provided five criteria for choosing parameters: (1) ease of understanding, (2) ease of estimation, (3) ease of calculation, (4) applicability of upper and lower bounds, and (5) other issues, e.g. assumptions, existing data, or need to be able to compare between different projects.

Suitable PDFs

Following the aforementioned five criteria in his operations research review of simulation, Williams (1992, p. 268) identified these common PDFs to be considered:

- Uniform: Should be used “only in special cases” due to its extreme simplicity;
- Normal (Gaussian): Is problematic because its 50% probability of negative value;
• Beta: Is “not easily understood” and also impractically requires four parameters;
• Triangular: Is easily understood and estimated, especially for a skewed function;
• Gamma: Requires complex calculations, but can useful for infinite upper bound;
• Other: For example Berny’s (1989) distribution, a modified Beta lacks clarity.

He found that “distributions... must be meaningful”; selection is more “psychological... than mathematical”, mean/mode must be distinguished, correlation should be modeled separately, and the Triangular “is... well-accepted” (Williams 1992, p. 269). Johnson (2002, p. 458) found that the Beta is “a suitable model..., as it provides a wide variety of distribution shapes over a finite interval”, but acknowledged that it is difficult to use. The Triangular can be an intuitive proxy for the Beta, because their differences are “seldom significant” (Johnson 1997, p. 387).

Studies also sought to identify appropriate PDFs in construction, including the Normal and Lognormal (Touran 1997), Beta (Schexnayder et al. 2005, Maio et al. 2000, Fente et al. 1999, Touran 1997,) and Triangular (Arízaga 2007, Back et al. 2000). Wilson et al. (1982) studied Beta versus Triangular, concluding that their differences were insignificant. Raymond (1999, p. 148) found a Triangular to be reasonable “and amenable to a mathematical formulation comparable with cost and schedule models and fast Monte Carlo analysis.” With use of more complex PDFs, e.g. Beta or Weibull, “little if anything is gained, and the intuitive simplicity of the triangular distribution is lost” (ibid. p. 149). Lee et al. (2013) benchmarked a CPM schedule and found a hierarchy of PDF conservatism: Exponential, Weibull, Uniform, Triangular, Lognormal, Normal, Beta (most to least). They noted that the same PDF is typically applied to all activities. Still it is possible to use different distributions within the same simulation to display differing characteristics of individual activities.

OTHER DISTRIBUTIONS AND METHODS

Program (Project) Review and Evaluation Technique (PERT)

Fazar (1962) defined PERT (also ‘Project’ Review and Evaluation Technique) as a dynamic tool to understand what must be accomplished to meet the time constraints of projects. “It is a system for diagnosing and anticipating the integrated influence of time, resources and technical performance on the outlook for achieving significant end objectives” (Fazar 1962, p. 598). Developed by the U.S. Navy Special Projects Office for its submarine Polaris nuclear missile program, this decision-making tool saved time by quantifying uncertainties where no knowledge exists (Fazar 1962). It has two elements: A ‘network flow diagram’ (a network schedule showing dependencies) and three ‘time expectations’ (Fazar 1962, p. 599): “The optimistic ($t_o$), the most likely ($t_m$), and the pessimistic time ($t_p$) estimate” for each activity within the schedule. Unlike Monte Carlo schedule simulation, PERT does not apply a PDF to determine a statistically-generated expected completion time ($t_e$). Rather, the expected duration ($t_e$) for each activity is hard-calculated as the sum of $t_o$ plus four times $t_m$ plus $t_p$, all divided by six. Fazar (1962) listed multiple outputs: (1) progress benchmarks, (2) uncertainty estimates, (3) activity criticality, (4) completion impacts, (5) rapid calculation, (6), graphical outputs for managers, and (7) future problem identification. Farnum and Stanton (1987, p. 289) showed that the subjective determination of optimistic and pessimistic completion times of a PERT analysis (i.e.
empirically determined) approximates the Beta, because the PERT expected time $t_e$ “closely approximates the actual relationship between the mean and the mode”.

The Tracy-Widom (T-W) Distribution

Unlike the above-referenced PDFs and CDFs that view statistical elements as acting independently (uncorrelated) and acted upon equally by outside events (risks and uncertainties), until the early 1990s no mathematical recognition or quantification of the various interactions between participants in systems existed. The little-known and yet-to-be applied Tracy-Widom distribution (Tracy and Widom 1992), which remains a topic of mathematical theorization, depicts the unique behavior of systems that move from mild to intense interactions (the critical intensity strength) among their participants, which cause a phase transition toward a different behavior. Such characteristics are also found in construction schedules, where a non-critical activity may be briefly delayed without affecting the project, or causing to overrun the deadline if it suddenly becomes critical and is delayed. It is hypothesized that T-W may be a better distribution to model construction schedule risk and uncertainty.

The lopsided distribution (Figure 1) drops off at a rate related to the number of eigenvalues $N$ (variables). On the left it diminishes more quickly as a function $N^2 \left(e^{-N^2}\right)$; on the right less quickly with $N \left(e^{-N}\right)$. It peak is $\sqrt{2N}$. In other words, larger values are more likely than shorter values when sampling. This matches the time extension-prone construction industry and may facilitate a more realistic model.

![Figure 1. Graphic Comparisons of Triangular, Beta and Tracy-Widom PDFs](image-url)
EXEMPLARS

To evaluate the effectiveness and applicability of the Tracy-Widom PDF, and following a process similar to Lee et al. (2013), the T-W, the Triangular, and Beta PDFs are applied to schedule networks of varying complexity to determine if the exaggerated skew and apex presents a materially better statistical representation of construction risk and uncertainty. The first exemplar schedule network is a fifteen-activity 72-day simple schedule from Lucko (2005) and Thompson (2012) as depicted in Figure 2. The second, a “J60” schedule, is from the Project Scheduling Problem Library (‘PSPLIB’) (Kolisch and Sprecher 1996), a complex sixty-two activity, 85-day network as depicted in Figure 3. Table 1 lists the results of 100 simulation runs.

Table 1. Statistics of T-W PDF versus Triangular and Beta PDFs

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Lucko (2005) (72 days)</th>
<th>PSPLIB ‘J60’ (85 days)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Triangular</td>
<td>82.08</td>
<td>3.83</td>
</tr>
<tr>
<td>Beta</td>
<td>88.70</td>
<td>5.30</td>
</tr>
<tr>
<td>Tracy-Widom</td>
<td>77.20</td>
<td>4.90</td>
</tr>
</tbody>
</table>

CONCLUSIONS AND FUTURE RESEARCH

Technology affords the construction industry the opportunity to model future work, whether it is process focused or time- and cost-based. Simon (1957) identified this need based on the limitations of the human mind for complex problem solving, and methodologies such as PERT and CPM have their origin around the same time. However, its application across the construction industry has yet to reach ubiquity, despite the wealth of Monte Carlo ‘add-ons’ to conventional analytical programs.

A requirement to model of risk and uncertainty is the probability distribution formula (PDF): The mathematical equation that statistically links potential outcomes with their probability of occurrence. There are a wealth of PDFs, with the most well-known being the Normal (Gaussian) distribution or ‘bell curve’. Each PDF has its
place and the key to their application is determining which most closely resembles the probability for risk and uncertainty to occur. The construction industry presents a unique situation, because most risks and uncertainty tend to extend project durations (i.e. are right-skewed), and their absence does not materially correspond to an equal reduction in duration (i.e. left-skewed). Rather, the absence of risk and uncertainty more regularly facilitate the project completion within the as-planned expectation.

There exist multiple camps with respect to advocating for the appropriate PDF for use in construction project simulation: Most surround the Triangular PDF and the Beta PDF, both of which can be right-skewed distributions. However, a new Tracy-Widom PDF is introduced. This distribution recognizes the collective behavior of the system, its interdependencies, and the extent to which system elements (participants, herein schedule activities) interact with one another – much like the sequential and concurrent or parallel dependencies that compose any CPM schedule network system.

Initial application of the T-W PDF when compared to the Beta and Triangular as depicted in Figures 3 and 4 indicate that the T-W shows promise for future use in construction schedule simulation. Its mean is closer to the as-planned duration and its variability (standard deviation) is within a similar range to the Beta and Triangular, yet it ranks among the largest duration range; but is values are shifted to the left.

![Figure 3. Modified Box and Whiskers Plot of Lucko (2005) Simulation Results](image1)

![Figure 4. Modified Box and Whiskers Plot of PSPLIB ‘J60’ Simulation Results](image2)

This shift in the range of values is caused by the T-W distribution being an unbounded (i.e. infinite) distribution, whereas the Beta and Triangular are not. Risk and uncertainty in the construction industry tend to remain unbounded, but do not materialize in all instances. Conversely, good fortune occurs, allowing activities and even projects to be completed ahead of planned duration. T-W facilitates this, but Triangular and Beta distributions do not, due to the limitations of a bounded PDF to simultaneously select all left skewed values to produce a better than as-planned result.

Future research on the T-W PDF should focus on applying this function \( a \ posteriori \) to an actual competed real-world construction schedule to determine if it more accurately models the conditions that such project experiences than Triangular and Beta PDFs, and see if its universality – its characteristic of accurately describing
correlated systems across multiple disciplines, where diverse “effects give rise to the same collective behavior” (Wolchover 2014) – holds for the construction industry.

REFERENCES


