Work-Path Modeling and Spatial Scheduling with Singularity Functions

Shabtai Isaac, Ph.D.\(^1\), Yi Su, S.M.ASCE \(^2\), Gunnar Lucko, Ph.D., A.M.ASCE \(^3\), David Dagan \(^4\)

\(^1\) Lecturer, Department of Structural Engineering, Ben-Gurion University of the Negev, Beer-Sheva, Israel, email: isaacsh@bgu.ac.il
\(^2\) Graduate Research Assistant, Construction Engineering and Management Program, Department of Civil Engineering, Catholic University of America, 620 Michigan Avenue NE, Washington, DC 20064, email: 61su@cua.edu
\(^3\) Associate Professor and Director of Construction Engineering and Management Program, Department of Civil Engineering, Catholic University of America, 620 Michigan Avenue NE, Washington, DC 20064, email: lucko@cua.edu
\(^4\) Ph.D. Candidate, Department of Structural Engineering, Ben-Gurion University of the Negev, Beer-Sheva, Israel, email: dudidag@gmail.com

Abstract

Current construction scheduling lacks an integrated planning approach that considers not just the time aspect of resources, but also the physical workspace within which they interact. Therefore, this research creates a mathematical model to support allocating time and workspace with concurrent activities and under safety constraints. A time-space chart represents crew movements along work-paths on a site with two dimensions; time is measured on the third axis. Total project duration is minimized by expressing work-paths as singularity functions. These range-based expressions are extended to the third dimension to determine distances between activities and
compact them. The approach ensures that temporal and spatial buffers between work-paths are maintained. It links scheduling with site layout planning and safety management, which allows generating a work plan that is both efficient to achieve and safe to execute. Model feasibility is validated by applying it to a case study of a major real-world construction project.

Keywords: Space use, paths, buffers, singularity functions, mathematical modeling

Introduction

The goal of this research is to increase the efficiency at which activities can be performed on a construction site. It will support decisions on allocating time and workspace to them without compromising the safety of workers. The importance of considering limited resources when planning construction projects has been widely recognized. Almost all projects are affected by limited availability and an appropriate resource allocation is essential (Fondahl 1991). However, resource constraints are not directly incorporated in many popular planning tools like the Critical Path Method (CPM) (Zhou et al. 2013). A noteworthy extension has been introduced by Kim and de la Garza (2003), who analyzed how actual float is less than it appears if laborers or crews are unavailable for a planned activity. Another important resource that must be considered is the available workspace. Space is almost always limited and may be required by several activities at the same time, which creates a risk of overcrowding (Zhou et al. 2013). Therefore, determining how to optimally allocate it among activities is needed to ensure both efficiency and safety.

Complexity of projects and the need to take multiple resources and factors simultaneously into account leads to mostly manually created project plans that are often inefficient. Bashford et al. (2007) for example documented on average about 75% idle time in building three residences.
Existence of such a significant idle time is of course also caused by technological constraints, not only by manual planning practices. A clear need exists for research on developing automated tools for construction planning, and specifically for resource allocation (Tauscher et al. 2014).

A tool that can support allocation of both time and workspace for a project requires a model that links the domains of scheduling, site layout planning, and safety management that provide crucial inputs for such a model. The objective of this research is therefore to derive a model that will contain an explicit representation of spatial, temporal, and safety constraints of a site. Such an integrated model complements existing planning methods. It could be used to solve problems that lie between domains, where lacking a comprehensive model they can only be solved by a laborious manual decision-making process. This can lead to ad-hoc and often suboptimal solutions and is especially true for large and complex projects. Complexity can manifest itself in the number of activities, relations between them, as well as in the physical size of the facility.

As this paper will elucidate, the model can be used to sequence concurrent activities that occur in different areas on the construction site and identify the optimal location for temporary storage, while ensuring that safe distances are kept between crews. Depending on the activities that they perform, work crews move along different paths on the site, as activities are executed repeatedly at different locations. All of these factors must be considered simultaneously. The model supports this by providing a representation of the spatial and temporal variables. The remainder of this paper is organized into the following sections: Literature Review presents previous relevant studies; Proposed Model: Goal, Assumptions, and Limitations introduces the goal, assumptions, and limitations of the proposed model; Model Definition provides a mathematical definition of the model; Model Implementation describes an implementation of the
model in a major real-world case study; and Contributions to the Body of Knowledge summarizes the accomplishments of this study and provides recommendations for future work.

**Literature Review**

Many studies on resource-constrained project scheduling exist, mostly outside the domain of construction management. These studies focused on optimization algorithms and their efficiency, as surveyed by Herroelen et al. (1998), who noted a prevalence of employing CPM assumptions and concepts in analytical approaches. Within construction project management, studies focused on site arrangement or spatially conflicting activities and can be sorted in six related categories:

1. Efficient utilization of site space during project execution is called *site layout planning*. Due to its impact on safety, productivity, and security, several site layout planning models were developed. Their common aim was generating the best layout for a set of constraints. An overview of site layout models was provided by Sadeghpour and Andayesh (2015).

2. *Topological taxonomies* organized the intricate spatial relations between different types of activities by capturing the spatial needs of activities to coordinate them. Needs were defined in terms of “shape, volume, and ability to overlap or stack multiple activities (Tommelein et al. 1992)” (Lucko et al. 2014, p. 133). Riley and Sanvido (1995) discretized areas and paths and presented various patterns of use, which may depend on the type of activity. They later (1997) described a space-considering planning approach to rank and prioritize decisions and identify conflicts. Echeverry et al. (1991) created some useful classifications of the physical interactions into ‘supported by’, ‘embedded in’, and others. Zouein and Tommelein (1993) used heuristics to overcome conflicts of spatial nature within a schedule. Akinci et al. (2002) ranked conflicts by importance, but continued to employ a limiting bar chart representation.
Spatial relations between planned activities to identify and prevent potential conflicts were introduced in computer-aided design (CAD) and decision-support systems (Akinci et al. 2002, Chua et al. 2010). The goal of these systems was to provide visualize the use of space on the construction site. Winch and North (2006) criticized that research has not focused sufficiently on workspace, and consequently developed a solution for identifying “available space on site, allocating activities to spaces, and optimizing space allocation in relation to the critical path” (Lucko et al. 2014, p. 133). Bansal’s (2011) tool connected a geographical information system (GIS) with a schedule that considered the spatial aspect. The present research overcomes limitations of previous work by these authors. Dagan and Isaac (2015, p. 64) had used “a matrix-based method for the definition of minimum safe distances between workers, and the use of 3D time-space diagrams to represent and analyze the dynamic movements of workers on site”. But that study implemented only spatial safety buffers, not any temporal ones. Its model was strongly limited to defining spatial buffers as the Euclidian distance between spatial coordinates of two workers on segmental paths. Moreover, it lacked a mathematical formulation of actual activities. Furthermore, it did not provide extensibility to the time and cost dimensions. Finally, it did not provide a scheduling algorithm. Lucko et al. (2014) had targeted a similar goal by using singularity functions to express rectangular work areas within a 3D coordinate system. That model also suffered from various limitations:

- Its areas had to be parallel to the two orthogonal spatial axes, which limited their shapes;
- It could not express any diagonal movement of workers across a workspace whatsoever;
- It could not express three-dimensional curves that are required to model paths with safety distances, but only ‘ramps’ and ‘steps’ as building blocks within its scheduling approach;
- It did not provide expressions for the projections of said shapes onto planes of two axes;
It did not give a formulation to calculate the closest proximity between two work paths at any location. This precluded the possibility of defining the required distance between two work paths according to their attributes, instead of treating all activities in the same way.

4. A number of computer implementations were developed that employed heuristics that considered spatial need in conjunction with sequential relations over time. Thabet and Beliveau (1994) used a linear schedule and also a CPM network in allocating workspace for limited resources. For this, they defined a space capacity metric of demand and supply on the site. Zouein and Tommelein (2001) pointed out that demand and supply are not necessarily constant, but may evolve as the project is executed and its physical shape and size evolve.

5. Some safety planning models attempted to consider where activities are located. Zhang et al. (2013, p. 183) extended a “commercially available BIM [building information model]” with their “rule-based system” for “automated hazard identification and correction” of a single type of accidents, falls, because it is the leading cause of fatalities. They were to be prevented by identifying edges and openings in a BIM model with an algorithm, which then suggested guardrails at such locations. Benjaoran and Bhokha (2010, p. 395) had previously developed a similar system “to automatically detect any working-at-height hazards”. Sacks et al. (2009) contributed to accident prevention by algorithmically creating scenarios of possible hazards and matched them with the probability of workers being in this location at inopportune times. Kang et al. (2011, p. 690), on the other hand, used a broader definition of risk factors, which encompassed “cost, duration, and dangerous condition of work site” within their CAD-based approach. Notable was the use of colors akin to a traffic signal to symbolize the level of risk.

6. Discrete event simulation (DES) is a specialized knowledge area, which models, analyzes, and optimizes construction tasks, processes, and operations. Its DES networks strongly differ
from network schedules, because their cyclic nature allows repetitions of steps, which may have probabilistic durations. Also, they model how resources get consumed by and released by activities (Martínez 2001). However, they cannot explicitly model actual space, unless a work area is modeled as a simple resource that is active or idle, i.e. occupied or empty space.

Despite their valuable contributions, these studies represented activities at a relatively high level and ignored detailed quantitative interactions. A comprehensive model is lacking that explicitly incorporates the intricate spatial, temporal, and safety constraints of construction activities.

Proposed Model: Goal, Assumptions, and Limitations

The goal of this research is to derive a model that will improve allocation of both time and workspace in a construction project. It will minimize the total project duration and the space that is occupied by the crews on the construction site. Since executing concurrent activities in close proximity can increase the risks for workers, safety constraints will also be taken into account. Safety requirements will be treated as non-negotiable hard constraints within the temporal-spatial environment, while other managerial aspects, e.g. the total project duration, are to be optimized.

Allocating time and workspace of course requires a model that explicitly represents spatial and temporal dimensions of the construction project. With such model, the progression of crews through the site can be carefully coordinated, ensuring that they perform their work in a way that is both efficient and safe. Safety constraints that require crews to be separated in time and space are modeled as temporal and spatial buffers. A three-dimensional (3D) time-space chart (Figure 1) will be used to represent the movements of different workers or crews. Horizontal axes are coordinates on the site, while the vertical axis measures time. Crew movements are 3D curves.
It is assumed that the model is established after an initial project plan has been prepared. This plan includes a detailed site layout, a schedule with planned activities, their locations on site, and a Preliminary Hazard Analysis (PHA) of the safety risks. It is further assumed that all activity durations have been planned in a consistent manner by the scheduler, and that they have not been subjectively inflated to include contingency for delays. These data are inputs for the model. It is applied to improve an existing plan, complementing tools that are currently used. Actions to improve the plan could include compacting the schedule down to the time buffers, adding time between activities if time buffers are violated, and identifying any violations of space buffers.

Limitations of the proposed model are that it will only consider a two-dimensional (2D) site layout where spatial relations exist in the horizontal plane. Relations between activities that are carried out at different heights can currently not be addressed by the model. Its implementation addresses only interior finishing activities. However, as will be explained in the conclusion, the authors are confident that its scope could be extended to additional types, e.g. earthmoving.

Workspace in Construction Projects

The proposed model represents the use of workspace over time by workers, who perform individual tasks that are typically repetitive and eventually compose unique activities within the work breakdown structure. Workspace here primarily refers to the areas that workers, materials, and equipment occupy for a certain period of time. It does not necessarily include paths along which workers move to reach those areas (e.g. from an elevator to the workspace). Paths that are used only momentarily or rarely are not explicitly modeled, as opposed to paths that are used intensively for an activity and therefore might interfere with other activities. The term work-path
means the succession of workspaces that workers use on a planar site (i.e. floor of a building). It is a general direction in which they progress, but not necessarily a precise location at all times.

In current practice, location-based schedules partially display the use of workspace over time. Also called linear scheduling, lines in a 2D diagram (Figure 2) of this visual technique represent activities at specific locations (Jongeling and Olofsson 2007, Russell et al. 2009). Points on the vertical axis indicate distinct locations within a structure, which are typically floors. The horizontal axis tracks time. Combining these data, it can be determined for any activity when it occurs where. A similar type has been used for infrastructure projects like roads or railroads. But here the horizontal axis measures distance along the length of project. Balancing productivities, called synchronization by Dagan and Isaac (2015) is desirable. An ideal linear schedule therefore is one whose activity lines are parallel and are separated only by their minimum required buffers.

Existing location-based diagrams are limited in their ability to show the actual progression of activities along different work-paths. They are either restricted to a physical division of the site into one type of repetitive units like floors of a building, or to a single path across the site (in the case of infrastructure projects). However, actual use of workspace is considerably more complex than that. Riley and Sanvido (1995) have demonstrated that different types of activities progress along different work-paths: Installing sprinkler pipes and ducts will progress along a linear path; assembling drywall will progress from one horizontal unit to another, whereas installing window panels may take a circular work-path along the perimeter of a building. Similarly, how heavy equipment moves on an infrastructure project is not necessarily restricted to a single linear path, but may occur in conjunction with other movements by workers or equipment. Existing location-based diagrams cannot provide a detailed representation of such a variety of work-paths. They are consequently strongly limited in terms of aiding in optimizing the allocation of workspace.
A 3D time-space chart will provide a more realistic representation of work-paths in this research. Instead of restricting the model to a limited number of repetitive units, e.g. sections of a building, or to a single linear path, the time-space chart (Figure 1) represents the locations of workers as coordinates on the horizontal axes at a given time on the vertical axis. Each work-path forms a 3D curve. As will be demonstrated, this chart can be used to increase efficiency via improved workspace allocation, while preventing crews from interfering with each other.

Buffers between Path Segments

The model explicitly represents temporal and spatial buffers that must be maintained between the activities on different work-paths, which constrain the resource allocation. Buffers are distances between two curves as measured on the respective axes. Both types of buffers must be taken into consideration to support having safe yet efficient construction sites with minimal waste. Buffers are required due to both the complexity of sites and the uncertainty that surrounds the activities.

Spatial Buffers

Spatial buffers are defined between work-paths to reduce safety risks whereby workers would be in proximity to hazards from concurrent activities. It is assumed that the level of risk depends on the distance from the hazards. Adherence to a predefined minimum distance between work-paths thus directly leads to reducing the risk. The distances that must be kept depend on the attributes of activities or tasks therein. In practice they are based on the footprint that a PHA has identified, e.g. the height of a component that may topple, the distance for a truck to fully brake, or how far the welding sparks may fly (Dagan and Isaac 2015). An opportunity also exists to define specific distances that must be maintained between two particular crews. Such a definition is based on the
assumption that a hazardous condition is created by both crews. For example, a welder can
accidentally cause a fire if the other specific task involves flammable materials like oil or paint.

Currently, spatial relations between activities are defined mostly as finish-to-start relations in
location-based schedules. Since these schedules are based on a particular order of working in the
zones or on a single work-path, such spatial relations can be converted into temporal relations. If
one crew thus has to remain at a certain distance behind another one on the same path, this can
be equivalently represented as a time lag between the pair. On the other hand, if two crews can
work on different paths, and must keep a certain distance between them at all times, then this is a
continuous spatial relation that cannot necessarily be defined by converting it into units of time.

Once the spatial buffers are defined, the actual distances between work-paths in the existing
construction plan can be automatically calculated using the time-space chart, and compared with
the required distances. This will allow the plan to be improved, by taking correcting actions such
as compacting the schedule up to the minimum required distances, or increasing the distance
between the crews in case their buffers are violated anywhere within the existing project plan.

Temporal Buffers

In addition to spatial buffers, temporal (time) buffers can be defined between different activities
that are performed along work-paths. In defining them, a distinction should be made as follows:

- **Lag:** The amount of time by which the start of a successor activity has to occur after the
  finish of a predecessor activity. It is often defined due to technological constraints;

- **Free Float:** The amount of time whereby a noncritical activity can be delayed without causing
  a delay to direct successor activities. It is an unintended byproduct of the scheduling process.
- **Contingency Buffer**: The amount of extra time that is allocated for an activity in excess of the minimum time that is required to complete that activity. It is done to ensure that unexpected delays will not affect successor activities. It is therefore usually allocated to critical activities; These three interstitial periods on the vertical time axis between a predecessor-successor pair of activities have different meanings in practice; lag and contingency are deliberate while free float arises as an artifact of creating the schedule. This research makes no judgement on the objective or subjective values of these items, but distinguishes them for completeness of this discussion.

Time buffers that are used in the model are composed of lag between activities (if it exists) plus a contingency buffer. The need for contingency buffers stems from the fact that construction projects are complex and fraught with uncertainty. In such environment, a tolerance is needed to absorb the impact of a deviation from the planned schedule before it affects the completion of the entire project. A contingency buffer may therefore be allocated to a noncritical activity only if it is required in excess of its free float. Representing time buffers in the model ensures that the improvement the efficiency of the project plan will not increase the risk of cascading delays. As has been mentioned, it is assumed that raw and adjusted durations can be clearly distinguished, i.e. that if a contingency is applied, it must be done in an objective and auditable manner to avoid potential abuses from manipulating durations. A detailed discussion of this issue of applying contingency in form of float or buffers in a fair and equitable manner to mitigate risk is beyond the scope of this research, but is the topic of ongoing research, as Lucko et al. (2016) described.

**Model Definition**

In the proposed model, the work-paths used by workers on the planar site are modeled as 3D curves. In the 3D coordinate system, the two horizontal axes $x_1$ and $x_2$ are the length and width of
the construction site, while the vertical axis $y$ is time. As noted in Proposed Model: Goal, Assumptions, and Limitations, the goal of the model is to minimize the total project duration and the space occupied by crews, while satisfying safety constraints. Specific research objectives of the model that are addressed in this section are:

- **Objective 1**: Expressing a 3D curve and 2D projections flexibly with singularity functions;
- **Objective 2**: Deriving an analogy for 2D algorithm steps to minimize total project duration, by reducing the time period between activities to the minimum temporal buffer, which is represented as a vertical distance between the 3D curves within the time-space chart;
- **Objective 3**: Determining how a single 3D diagonal line can be offset perpendicular to its direction (as seen in the $x_1$-$x_2$-projection) by a specified amount to define horizontal spatial buffers between paths that fulfill safety distance requirements to other ongoing activities;
- **Objective 4**: Identifying and calculating the closest proximity between two 3D diagonal lines not just vertically, but also horizontally within the $x_1$-$x_2$-projection, to simultaneously address both safety distance and time efficiency requirements through a minimization approach.

### Singularity Functions

Temporal (time) buffers are vertical distances between curves; spatial buffers are horizontal ones. To measure the minimum distances, the curves will be mathematically modeled as singularity functions. The traditional definition of singularity functions links one independent variable and one dependent variable per Equation 1. For consistency with previous publications, the variables are designated $x = \text{work quantity (or the space within which it is performed)}$, $y = \text{time}$, $z = \text{cost}$.

\[
y(x) = s \cdot (x - a)^n = \begin{cases} 0 & \text{for } x < a \\ s \cdot (x - a)^n & \text{for } x \geq a \end{cases}
\]

**Eq. 1**
Where the factor $s$ is a strength, the cutoff $a$ is an activation, and the exponent $n$ is a behavior. For this 2D definition, a complete algorithm for linear schedules of $y(x)$ has been created, which minimizes total project duration subject to all buffer constraints and calculates criticality and float (Lucko 2009, Lucko and Peña Orozco 2009). It first ‘stacked’ activities and their buffers sequentially along the time axis with conservative finish-to-start links and then ‘consolidated’ them by overlapping activities as far as their buffers allowed to shorten the overall schedule. Recent extensions to modeling also resources are omitted; cash flow is briefly discussed below.

Mathematical rules apply. A step has exponent zero, a slope has exponent one. Singularity functions are right-continuous and remain active until positive infinity. Deactivating a behavior thus means subtracting an appropriate term at a later cutoff. Actual activity start and finish dates may differ from the planned ones: They may be postponed by a shift $d_1$ of the entire activity. Finishes may additionally be impacted by an internal delay $d_2$ within the duration $D$ itself. Actual dates may be indicated with an asterisk: $a_s^* = a_s + d_1$, $a_f^* = a_f + D + d_1 + d_2$. Equation 2 provides the general form of a 2D progress curve of an activity $i$ that adds the work quantity $W$ to existing work $a_s$, starting at time $y_s$ with an uninterrupted productivity that is the ratio $\Delta y / \Delta x = W / D + d_2$. If interruptions are planned or occur, they can be modeled by adding step terms to Equation 2.

\[
y(x) = y_s \cdot (x - a_{si})^0 + \frac{W}{D + d_2} \cdot [(x - a_{si})^i - (x - a_{ri})^i] - (y_s + D + d_2) \cdot (x - a_{ri})^0
\]

Eq. 2

This research improves the attempt of a 3D model by Lucko et al. (2014), whose horizontal axes $x_1$ and $x_2$ were length and width and the vertical $y$ was time. They had multiplied two singularity functions with different independent variables. This represented a projection of a shape in the $x_1$-$y$ plane multiplied by a projection in the $x_2$-$y$ plane. In other words, the general principle $y(x_1, x_2) = y(x_1) \cdot y(x_2)$ was applied. For example, a block and a ramp (that is sloping in the $x_1$-$y$ plane), both of final height $y_h$, had been represented by Equations 3 and 4, respectively.
\[
y(x_1, x_2)_{\text{block}} = y_h \cdot \left( (x_1 - a_{s1s})^0 - (x_1 - a_{s1F})^0 \right) \left[ (x_2 - a_{s2s})^0 - (x_2 - a_{s2F})^0 \right]
\]
\[
y(x_1, x_2)_{\text{ramp}} = y_h \cdot \left[ (x_1 - a_{s1s})^1 - (x_1 - a_{s1F})^1 - (a_{s1F} - a_{s1s}) \cdot (x_1 - a_{s1F})^0 \right] \left[ (x_2 - a_{s2s})^0 - (x_2 - a_{s2F})^0 \right]
\]

Eq. 3

Eq. 4

However, their model had several conceptual problems that limited its analytical capabilities:

- The model was imprecise, since its singularity functions were defined as right continuous. At all three ‘far’ corners (seen from the origin) of a 3D shape, the value \( y(x_1, x_2) \) would already fall to zero. Therefore, this manuscript proposes the use of adding a left-continuous definition per Equation 5, which the index \( R \) or \( L \) expresses. At a far corner \( x = a \), \( y(x)_R \) would already give zero, but \( y(x)_L \) still gives a nonzero value. This distinction solves corner points properly.

- The model was unrealistic in that it allowed only behavior acting parallel to the \( x_1-y \) and \( x_2-y \) planes onto which it must be projected. Combining planes multiplicatively is problematic because some 3D curves cannot be modeled by two simple 2D projections. For example, viewing a sloping edge head-on one cannot distinguish if it slopes away from or toward the viewer. Basing a 3D curve on an \( x_2(x_1) \) plane would create new problems, being merely a \( y(x_1) \), not a true \( y(x_1, x_2) \) model. In other words, diagonal movements across the \( x_1-x_2 \) floor could not be modeled, which severely impeded expressing realistic work-paths. This problem stemmed from multiplying two 2D shapes for a four-cornered 3D volume, e.g. a block from two rectangles, a ramp from a triangle and a rectangle, and a pyramid from two triangles.

\[
y(x)_L = s \cdot (x - a)_L^n = \begin{cases} 0 & \text{for } x \leq a \\ s \cdot (x - a)_L^n & \text{for } x > a \end{cases}
\]

Eq. 5

**Extension to 3D Schedule Compaction**

Due to the limitations of previous 3D singularity functions, a new approach is systematically developed. Experiments with different shapes and constellations were performed with
calculations on how two 3D activities could be properly added and subtracted (both of which are
needed for the stacking and consolidation to gain a complete schedule) and what the meaning of
the result was – a vertical distance, area, or volume between the two curves. An insight gained
from these experiments is the necessity of being able to identify the intersection of two 3D
curves in the $x_1$-$x_2$ plane. A synchronization function of Equation 6 is accordingly derived. It
contains the relationship between $x_1$ and $x_2$. The product of the two factors in Equation 6 equals
one when the point is on the line defined by $x_2(x_1) = x_2$ (because only when $x_2(x_1) = x_2$, $(x_2(x_1) -
\cdot x_2)^0$ and $(x_2 - x_2(x_1))^0$ are both 1).

\[
x_2(x_1)_{\text{synchronization}} = (x_2(x_1) - x_2)^0 \cdot (x_2 - x_2(x_1))^0
\]

Eq. 6

Applying this synchronization function to two traditional functions in the $x_1$-$y$ and $x_2$-$y$ planes
per Equation 7 overcomes the aforementioned problem of a four-cornered 3D volume when a 3D
curve is needed. Another consideration is that the slopes or steps of the two multiplied functions
would give an incorrectly large strength factor. Therefore their square root is taken. For negative
values an absolute value operator $|$ $|$ removes the sign so that the root can still be evaluated.

\[
y(x_1, x_2)_{\text{synchronized}} = (x_2(x_1) - x_2)^0 \cdot (x_2 - x_2(x_1))^0 \cdot \sqrt{y(x_1) \cdot y(x_2)}
\]

Eq. 7

A pair of the former 2D function of Equation 2 is used and called projection functions per the
slightly relabeled Equation 8 (here only shown for $x_1$, analogous also for $x_2$) to indicate that each
shows a 2D view, whose multiplicative combination per Equation 5 gives the full 3D behavior.

\[
y(x_1) = [a_{sF} \cdot (x_1 - a_{sLS})]_R + \frac{a_{sF} - a_{sS}}{a_{sLF} - a_{sLS}} \cdot [(x_1 - a_{sLS})_R - (x_1 - a_{sLS})_L] - a_{sF} \cdot (x_1 - a_{sLF})^0
\]

Eq. 8

- Note that Equation 8 contains both right- and left-continuous singularity functions. For starts
  (closer to the origin at the left edge of a shape) their $R$ form is used; for finishes (at its right
  edge) their $L$ form. Example 1 demonstrates this newly derived concept. Assume two
activities with the sequence \( A \to B \), each of which is a bounded short straight line segment. Their \( x_1, x_2, y \) start and finish coordinates are \( \{2, 1, 1\} \) to \( \{3, 4, 3\} \) for \( A \) and \( \{1, 2, 8\} \) to \( \{4, 3, 6\} \) for \( B \). As Figure 3 shows, they run in different directions at different heights in an overlapping range on the \( x_1-x_2 \) plane (their shared footprint). The dashed line is the buffer of the predecessor \( A \) that the analysis considers in the calculation. In their current constellation, these activities do not touch. Basic questions that must be answered to accomplish previously established research objectives are (a) where are they closest and (b) what is their distance?

Coordinates \( \{a_{x1S}, a_{x1F}, a_{x2S}, a_{yS}, a_{yF}\} \) for the singularity functions of \( A \) and \( B \) thus are \( \{2, 3, 1, 4, 2, 3, 8, 6\} \). Activity \( A \) is written as the pair of projection functions of Equations 9 and 10, activity \( B \) is modeled analogously.

Floorplan projection functions can also be created, giving Equation 11 and analogous for \( B \).

Equation 12 applies the synchronization of Equation 7 to both \( A \) and \( B \). Evaluating it for any nonzero \( y(x_1, x_2) \) identifies their intersection in the \( x_1-x_2 \) plane as shown in Figure 3c.

Evaluating Equation 12 yields a nonzero value only for exactly \( \{x_1, x_2\} = \{2.5, 2.5\} \), which is found by implementing the equation in spreadsheet software. Its individual components are the
3D coordinates of \{2.5, 2.5, 2\} for A and \{2.5, 2.5, 7\} for B. The proximity \(y(x_1, x_2)_{\min(B-A)} = 5.0\) is found in the same calculation. This is the minimum vertical distance where the two work-paths cross in the floorplan. This distance can be compared with the required minimum time buffer \(\Delta t\) between the activities that are carried out on those paths, to ensure that this time buffer \(\Delta t\) is not violated. This solves questions (a) and (b) in the 3D extension of the so-called ‘stacking’ in the 2D algorithm for linear schedules (Lucko 2009). The distance of 5.0 - \(\Delta t\) must be subtracted from the intercepts of both projections of \(B\) on the vertical time axis to give Equations 13 and 14. This lowers \(B\) down until it just touches \(A\)’s time buffer. Note that this shift does not make any changes on the other two axes. Figure 4 shows the result with time buffer \(\Delta t = 1\), which concludes the ‘consolidating’. Note that the 2D projections of Figure 3 would incorrectly imply that \(B\) only must be lowered by \(y = 3.67\). However, in 3D it would not touch \(A\)’s time buffer. Only the calculation guarantees that it occurs. This fulfills Objectives 1 and 2: New equations can flexibly express a 3D curve and it three 2D projections in its coordinate system. An analogy for the stacking and consolidating steps of the 2D algorithm (Lucko 2009), which has been briefly summarized above, has been derived for 3D temporal-spatial scheduling problems, which can be used to minimize the total project duration.

\[
y(x_1)_B = (8 - (5 - \Delta t)) \cdot \langle x_1 - 1 \rangle^0_K + \frac{6 - 8}{4 - 1} \cdot \left[\langle x_1 - 1 \rangle^1_K - \langle x_1 - 4 \rangle^1_L\right] - (6 - (5 - \Delta t)) \cdot \langle x_1 - 4 \rangle^0_L \tag{Eq. 13}
\]

\[
y(x_2)_B = (8 - (5 - \Delta t)) \cdot \langle x_2 - 2 \rangle^0_K + \frac{6 - 8}{3 - 2} \cdot \left[\langle x_2 - 2 \rangle^1_K - \langle x_2 - 3 \rangle^1_L\right] - (6 - (5 - \Delta t)) \cdot \langle x_2 - 3 \rangle^0_L \tag{Eq. 14}
\]

Extension to 3D Paths for Spatial Scheduling

More challenges stem from the desired use to model and optimize paths within a 3D space-time schedule, where \(x_1\) and \(x_2\) are length and width on a site and \(y\) is time. It requires two extensions:
Note that **Objective 3** adds a left-and-right safety buffer on both sides of a 3D activity line, offsetting it by equal amounts. It can be visualized as three parallel lines (dashed, solid, dashed).

Whereas a vertical time buffer would only require a $\Delta y$ value in its intercept as compared to its host activity line; horizontal safety buffers are more complicated, because they change both $x_1$- and $x_2$-coordinates. A set of basic relations must be derived for this objective. Measured perpendicular to the direction in the floorplan, a horizontal safety buffer is $D$ wide, where

$$\Delta x_1^2 + \Delta x_2^2 = D^2$$

holds. Figure 5 shows this for a generalized sloped line of an activity and its horizontal safety buffer within their 2D coordinate system. By geometric comparison it is found that angle 1 that exists between the activity and the $x_1$-axis is the same as angle 2 between the buffer and the $x_2$-axis. Its tangent is the opposite leg divided by the adjacent leg,

$$\tan(1) = \frac{\Delta x_2}{\Delta x_1}$$

This ratio is the same for activity and horizontal safety buffers. Inserting

$$\Delta x_1 = \frac{(a_{s2F} - a_{s2S})/ \left( a_{s1F} - a_{s1S} \right)}{\left( a_{s2F} - a_{s2S} \right)/ \left( a_{s1F} - a_{s1S} \right)}$$

in

$$\Delta x_1^2 + \Delta x_2^2 = D^2$$

gives

$$\Delta x_2^2 \cdot (S^2 + 1) = D^2$$

Taking the square root gives

$$\Delta x_2 = D/\sqrt{S^2 + 1}$$

and finally. Thus $\Delta x_1 = \Delta x_2 \cdot S$, which is intuitively correct, confirming this derivation. It will be called $S_{x1x2}$ to distinguish it from slopes within the other projections, $S_{x1y}$ and $S_{x2y}$. Using these, Equation 15 derives the $x_1$-$x_2$-projection (floorplan view) of horizontal safety buffers. Its ‘±’ terms describe offsets; the ‘–’ is safety buffer 1 to the right if the activity grows into positive $x_1$, $x_2$, $y$ directions.

$$x_2(\Delta x) = (a_{s2S} + \Delta x_2) \cdot (x_1 + \Delta x_1) - a_{s1S} + S_{x1x2} \cdot \left( (x_1 + \Delta x_1) - a_{s1F} \right)_R - \left( (x_1 + \Delta x_1) - a_{s1F} \right)_L$$

$$- (a_{s2F} + \Delta x_2) \cdot (x_1 + \Delta x_1) - a_{s1F} )_L$$

Eq. 15
This is only the $x_1$-$x_2$-projection. Projections onto the $y$-$x_1$ and $y$-$x_2$ planes can be analogously written (inverting the ‘±’ sign as needed), but are omitted for brevity. In Example 2, offsetting line $A$ of $\{1, 1, 1\}$ to $\{4, 4, 4\}$ sideways by $\Delta x_1 = \Delta x_2 = 1$ (width $\sqrt{2}$) gives safety buffers of $\{2, 0, 1\}$ to $\{5, 3, 4\}$ to the right and $\{0, 2, 1\}$ to $\{3, 5, 4\}$ to the left, as can be verified on paper. But to fulfill Objective 3, this must be calculated. Inserting offsets into Equation 15 gives Equation 16.

$$x_2(x_1)_{\text{Buffers}} = (1 \mp 1) \cdot (x_1 \mp 1) + \frac{4 - 1}{4 - 1} \cdot \left(\left((x_1 \mp 1) - 1\right)_R - \left((x_1 \mp 1) - 4\right)_L\right) - (4 \mp 1)$$

Eq. 16

Using all minuses gives $x_2(x_1)_{\text{Side Buffer}} = 0 \cdot (x_1 - 2)_R + 1/1 \cdot \left((x_1 - 2)_R - (x_1 - 5)_L\right) - 3 \cdot (x_1 - 5)_L$ correctly for safety buffer 1. Safety buffer 2 and projections are analogous. This satisfies Objective 3. For fulfilling Objective 4 the closest proximity must be calculated. Example 2 is broadened with another activity $B$ of $\{1, 4, 4\}$ to $\{4, 1, 4\}$, which has been deliberately constructed so that the proximity is not parallel to any of the planes, i.e. takes on the most difficult imaginable case. In this case, the Euclidian distance in 3D space has an odd unit, the square root of ($\text{meter}^2 + \text{meter}^2 + \text{hour}^2$), which is less intuitive to understand than its projections on the axes in meters or hours. A method is provided for calculating it separately in space and time units, which would be used in the future research. Two steps are necessary once again, (a) where the proximity occurs and (b) what its distance is. After reviewing vector algebra, a singularity vector is defined inspired by the point-direction-form of traditional vectors. Equations 17 and 18 set start coordinates of $A$ and $B$, plus a variable $h$ that runs from $\{0 \text{ to } 1\}$ as an activity progresses toward its finish. Note that the second term contains the difference of finish minus start coordinates.
The proximity \( P \) between \( A \) and \( B \) is expressed by Equation 19, which Equation 20 simplifies.

\[
P_{ab} = \left[ (B_{x1} - A_{x1})^2 + (B_{x2} - A_{x2})^2 + (B_y - A_y)^2 \right]^{1/2} = \left[ \left( \langle h_a - 0 \rangle^0 - \langle h_a - 0 \rangle^0 \right)^2 + 3 \left( \langle h_a - 0 \rangle^1 - \langle h_a - 1 \rangle^1 \right)^2 + 3 \left( \langle h_a - 0 \rangle^1 - \langle h_a - 1 \rangle^1 \right)^2 \right]^{1/2} \]

\[
P_{ab}^2 = \left[ \left( (1 - 1) + 3 \cdot h_a - 3 \cdot h_a \right)^2 + (\langle h_a - 0 \rangle^0 - \langle h_a - 1 \rangle^0)^2 \right]^{1/2}
\]

First-order and second-order partial derivatives with respect to \( h_A \) and \( h_B \) help identify where the closest proximity occurs per Equations 21 and 22 (analogous for \( \partial / \partial h_B \) that is omitted here):

\[
\partial / \partial h_A \left( \frac{\partial}{\partial h_A} P_{ab}^2 (h_a, h_b) \right) = 2 \cdot (3 \cdot h_a - 3 \cdot h_a) \cdot (-3) + 2 \cdot (3 - 3 \cdot h_a - 3 \cdot h_a) \cdot (-3) + 2 \cdot (3 - 3 \cdot h_a)
\]

\[
(-3) = -18 \cdot h_a + 18 \cdot h_a - 18 + 18 \cdot h_a + 18 \cdot h_a - 18 + 18 \cdot h_a = -36 + 54 \cdot h_a
\]

\[
\frac{\partial}{\partial h_A} \left( \frac{\partial}{\partial h_A} P_{ab}^2 (h_a, h_b) \right) = 54 > 0
\]

Setting each first-order partial derivative to zero yields \( 0 = -36 + 54 \cdot h_A \) and \( 0 = -18 + 36 \cdot h_B \), which gives \( h_A = \frac{2}{3} \) and \( h_B = \frac{1}{2} \) to indicate the points of the closest proximity. Entering them into Equations 17 and 18 gives the points as \( \{3, 3, 3\} \) and \( \{2.5, 2.5, 4.0\} \). Finally, inputting them into Equation 20 gives the proximity (in odd units) as \( P_{ab}^2 = (3 \cdot h_a - 3 \cdot h_a)^2 + (3 - 3 \cdot h_a - 3 \cdot h_a)^2 + (3 - 3 \cdot h_a)^2 \)
\begin{align*}
&(−1/2)^2 + (−1/2)^2 + 1^2 = 1.5 \text{ so that } P_{ab} = \sqrt{3/2}. \text{ Adding a condition that the vertical components} \\
&\text{should be identical, i.e. } A_y = B_y \text{ or } B_y - A_y = 0, \text{ would give just a horizontal distance. For Example} \\
&2, \text{ this gives } 0 = (4 - 1) \cdot 3 \cdot h_A \text{ from Equation 20, i.e. } h_A = 1 \text{ and the partial derivative for } h_B \text{ is} \\
&\text{unchanged, so that } h_B = \frac{1}{2}. \text{ Validation in a computer and a paper model confirm this as correct.} \\
&\text{The closest proximity in the } x_1-x_2\text{-plane thus occurs between the finish of } A \text{ and the middle of } B. \\
&\text{These calculations complete Objective 4. It remains to implement the entire model for validation.} \\
\end{align*}

\textbf{Model Implementation}

To validate the model it was implemented retroactively in a case study of a real-world project, in which a large manufacturing plant for electronic devices was remodeled and expanded to accommodate a new manufacturing technology. The project for one of the largest industrial firms in the world has a two year duration and costs billions of dollars. Of this, some $800 million was allocated to construction and the remainder to production machinery. Consequently, no expenses were spared when it came to planning the project. The project was planned by expert planners, who worked for one of the largest construction companies in the U.S., which prides itself on implementing a ‘zero accidents’ policy. The project included removing the existing machines and their infrastructure, executing structural and architectural changes to the manufacturing hall itself, and installing new mechanical, electrical, and plumbing (MEP) systems and production machines. Its owner’s name and location are withheld in this paper due to the proprietary nature.

To create a case study, the execution of this project was followed for a period of two months, and a variety of relevant project data were collected and analyzed. During this period, crews of workers who performed activities in the production hall were closely observed. These crews belonged to different subcontractors for MEP and architectural disassembly works. The number
of workers in each crew, their locations throughout a workday, and the progress of the activities were observed and documented. The observation period covered different disassembly activities following the removal of the existing machines. These activities included the disassembly of fire protection systems, raised floors, dropped ceilings, and heating, ventilation, and air conditioning (HVAC) components like ducts and filters. The site was divided into narrow service ‘areas’, labeled 1A, 1B, 2, 3, and 4, where most of the ducts and pipes were installed, plus rectangular production ‘sectors’ A-F, where the production machines were installed (Figure 6). Some components (ceiling modules and HVAC filters) were packed after disassembly and temporarily stored in predefined sectors in the manufacturing hall before removal to a permanent storage site.

During the period when their activities were observed, crews moved through areas in a predefined order from area 1A to 1B, 2, and finally 3. Within each area, they moved along paths per the coordinates of Table 1. While area 4 appears in Figure 6, no activities that were selected for analysis of the case project occurred there. In the computer implementation, the remaining actually used areas 1A and 1B, 2, and 3 are discretized into seven individual rectangular areas.

Activities that were carried out by different crews were sequenced per predefined finish-to-start relations. A minimum horizontal safety buffer of 8 meters had to be kept between adjacent work-paths to accommodate the use of larger equipment and to provide a safe work area around it at all times. HVAC filters that were disassembled were temporarily stored in sector F, while the ceiling modules were temporarily stored in sector D. They were later removed from the hall, along the diagonal paths in Figure 6, in a single batch and over a period of 4 work days in order to clear space for other activities. For brevity, the activity name ‘Remove D to E’ that is used in the following means in full ‘Remove Modules from Areas and Store in D and Exit at E’ and the analogous ‘Remove F to C’ means ‘Remove Filters from Areas and Store in F and Exit at C’.
Following the removal of the filters and modules, four other activities were carried out in each sector to disassemble components from the dropped ceiling and the raised floor. Each activity was carried out in parallel by two crews. One crew worked in sectors D, F, and B, in that order, and the second crew worked in sectors E, C, and A. Note that the sequence here is first removing filters and modules in the areas and then the activities conducted in the sectors. Crews in sectors D, F, and B and sectors E, C, and A have already taken activities in areas 1A and 1B, 2, and 3 into account, because they are already finished. Coordinates of the start and end points of paths are relative to the lower left corner of the site plan. Table 2 lists their actual progress.

Note that the four activities occurred sequentially along each respective path. Durations depended on the total path length and progress rates, which differed between areas and sectors. For example, the durations of the four activities in area 1A were 4.8 days, 4 days, 6.9 days, and 8 days. Their profiles are the quadruple lines with different slopes in the dashed ellipse in Figure 7. Each sector is subdivided into multiple parallel work paths for the four activities. Their relatively large progress rates allow rather short durations; their profiles in Figure 7 are a tight cluster. Four activities can be distinguished in the areas, but are too close to label individually in the sectors.

**Establish Model**

Entering the case study data into the general projection function per Equation 8 for both \( y(x_1) \) and \( y(x_2) \) and applying the synchronization function per Equation 7, the total duration of the project is 161.3 days. Note that if a path is parallel to the \( x_1 \)-axis, the equation is updated per Equation 23, and vice versa if parallel to \( x_2 \). The time-space chart in Figure 7 allows several observations: First, each thin line is one work-path, i.e. a crew trajectory. This helps checking for any conflicts or collisions among concurrent crews. Second, parallel work-path bundles within the same area
or sector, e.g. as circled, follow a finish-to-start relation. Third, the distance between paths can easily be changed via a model parameter to generate improvements from a denser arrangement. Note that it is assumed that no temporal overlap exists between repeated different activities along a single work-path, because they are performed by the same crew. This also allows objective comparisons between using one or multiple crews, which is investigated in the following section.

\[
y(x_1, x_2) = \left( x_2(x_1) - x_2 \right)^0 \cdot \left( x_2 - x_2(x_1) \right)^0 \cdot \sqrt{y(x_1) \cdot y(x_2)} = \left( x_2(x_1) - x_2 \right)^0 \cdot \left( x_2 - x_2(x_1) \right)^0 \]

\[
\left[ a_{ys} \cdot \left( x_1 - a_{x1S} \right)_R^0 - \left( x_1 - a_{x1S} \right)_L^0 \right] \\
\cdot \left[ a_{ys} \cdot \left( x_2 - a_{x2S} \right)_R^0 + \frac{a_{yF} - a_{yS}}{a_{x2F} - a_{x2S}} \cdot \left( x_2 - a_{x2S} \right)_R^1 - \left( x_2 - a_{x2S} \right)_L^1 \right] - a_{yF} \cdot \left( x_2 - a_{x2F} \right)_L^0
\]

Eq. 23

**Schedule Improvement: Changing Number of Batches and Temporary Storage Locations**

Arguably the most advanced planning method that project planners may use to attempt handling time and space is the linear scheduling method (LSM). But LSM can only generate solutions for separating activities that are executed along a single linear route, or within a set of discrete work areas. But work in this project was carried out in parallel along multiple paths within 2D areas. Due to the spatial and temporal complexity of coordinating the different concurrent activities on site, and lacking the new model that has been developed in this study, the planners of the project could not perform calculations like those that are described below to further improve their plan. Instead, they had to rely on ad-hoc solutions and their experience, to plan the removal activities.

During execution of *Demo Filter and Modules*, the demolished items were temporarily stored in two places; filters in sector F, modules in D. To test the model, it was used to analyze whether the project duration could be reduced by changing the way in which the filters and modules were...
removed from their temporary storage places and off the site. It was speculated that by using the model, a potential improvement of the schedule could be realized by changing the number of batches in which the demolished filters and modules would be removed from sectors D and F, where they were temporarily stored. Table 3 lists quantities of filters and modules for demolition.

A parallel sequence of activities exists in sectors D → F → B and E → C → A. Sectors D and E are adjacent to each other, therefore one must consider a conflict when the work-path of removing modules from section D crosses sector E and area 1B. Assuming that all filters and modules are removed in one batch after Demo Filter and Modules is finished in areas 1A, 1B, 2, and 3, it will take 4 days for removal, which postpones subsequent activities in sectors D-F-B and E-C-A (since their first activities in D and E are enabled by the removal of modules from D). If the filters and modules are removed in two batches instead of one, the removal of filters from sector D can be scheduled in proportion to the total quantity of the demolished filters in areas 1A, 1B, and 2. Per Table 3, the first batch will be removed when half of the Demo Filter and Modules activity is finished (within 1B). The model is used to analyze whether removing modules from sector D will affect the execution of activity Demo Ducts in area 1B. Removing modules and filters in either 2, 3, or 4 batches is analogous. Figures 7 through 10 are time-space charts for different numbers of batches with removal work-paths in red. Using the model, it was found that increasing batches to 3 reduces the duration of the project without a higher safety risk.

In addition to changing the number of batches in which modules and filters are removed, the selection of possible alternative temporary storage locations might further improve the schedule, since removal of components in additional batches would reduce the storage pile. Smaller piles can be stored in a smaller sector closer to the exit, which in turn reduces the duration of removal activities. For example, with the same crew at the same speed, but from another storage location,
the duration for removing two batches can be reduced from 4 to 2.2 days (Table 4). Using the model, the optimal solution was found to be the storage of the modules in sector E instead of D, of the filters in sector C instead of F, and their removal in 4 batches from each location.

A reduction of 3.5 days was achieved with the model compared to the original schedule. It did not have to resort to increasing productivities or safety risks. This improvement in the total project duration was created retroactively and unfortunately could not be actually implemented in the project. The original schedule as presented in this paper corresponded to the actual on-site progress, based on the observations that were made. In defense of the project planners, it could have been considered satisfactory, given the currently available planning methods. However, as this research has shown, it could have been improved further with the new modeling approach.

While this improvement may seem relatively modest, it should be noted that it was achieved after the original plan had been carefully prepared by highly experienced experts. In addition, the improvement was the result of one small change in a specific aspect of the project that served as an illustration of the new approach. It is not claimed that this is the only improvement that would have been possible on this project. In fact, more changes could be explored in its other activities, which would exceed the scope and length of this paper. Implementing the new approach on other projects will of course yield different results, whose magnitude depends on their time and space constraints and also the efficiency that planners are able to achieve within the original schedules.

### Schedule Improvement: Considering Cost Information

Cash flow management is vital for construction project management. For the aforementioned schedule improvements, the cost of varying the number of batches for the activities of the project part that was selected is analyzed exemplarily to derive its impact on the financial performance
of the overall project. Table 5 shows their planned cost data, including indirect and direct cost, markup for overhead and profit, owner retainage, financing interest, and credit limit (set at 75% of total cost). Labor cost was $200 per worker-day and the numbers of workers were 4 for Demo Fire Protection, 3 for Open Plenum Bolts, 25 for Demo Filters and Modules, 10 for Demo Ducts, 6 for Remove Filters and Modules, 8 for Demo Filters, 10 for Demo PEDs/Metal Tiles, and 8 for Demo Stanchions, and 8 for Repair Flooring. These cost data are entered into an already existing ‘synthetic cash flow model’ by the authors (Su and Lucko 2015a, 2015b). Said model calculates cost, pay, and balance at any point in time during the project and considers the time value of money and periodic phenomena, e.g. monthly payment and charging interest. Interested readers are referred to the detailed description of cash flow analysis in these earlier publications, as brevity precludes repeating that already well-documented application of singularity functions.

The cash flow of this project part with one batch is shown in Figure 11. Figures for different numbers of batches and storage locations are similar and omitted for brevity. This cost analysis has several benefits: First, it uses the same pattern of equations as the schedule equations of the new model, except that the multiplicative factor for progress slope is simply replaced with a cost slope (i.e. cost divided by duration). This allows seamlessly adding cost to the schedule model. Second, the cash flow portrays the growth path of the project in terms of its cost, which allows evaluating the project performance (here exemplarily for the selected MEP activities) for both the dimensions of time and cost. For example, the original plan had a credit limit at 75% of the total cost of $544,510, which would be $408,383. But the calculation reveals a maximum negative balance of only -$184,106 per Figure 11. Thus the planned credit limit based simply on a percentage exceeded the actual need and could be reduced to save potential financing fees.
A unique feature of this case study project is its massive value of approximately $10,000,000 per day. Thus cost increases from analyzing the cash flow performance of different batches and storage locations per Table 6 are very small compared to the extremely substantial savings that can be realized for every day of finishing the project early. Here the schedule had absolute priority and cost considerations at such a small scale were essentially found to be insignificant.

Contributions to the Body of Knowledge

The contributions of this research to the body of knowledge in construction management include:

- It establishes an integrated model that links scheduling, site layout planning, and safety management to augment existing planning methods toward solving inter-domain questions;
- Its model explicitly and comprehensively incorporates intricate spatial, temporal, and safety constraints of construction activities, and represents the detailed quantitative interactions between concurrent activities on a site. This allows carefully coordinating the crew progress through a site, ensuring that they perform their work in a way that is both efficient and safe;
- The mathematical modeling of the work-paths of crews as singularity functions, supporting an improved allocation of both time and workspace in a construction project, while taking into consideration both temporal and spatial buffers between different work-paths;
- Its new approach can minimize the total project duration while obeying safety requirements of minimum time and space distances between adjacent work paths in their 3D environment without having to resort to increasing productivities or even assuming additional safety risks.

To validate the feasibility of the model it was implemented in computer code to improve the planning of a real-world project, demonstrating that it can successfully resolve a scheduling problem that would require considerable effort to solve manually without any formal method.
Recommendations for Future Research

Newly possible, but beyond the scope of this paper, is to perform a time-space tradeoff analysis akin to the well-known time-cost tradeoff for network schedules. Adding even more planning dimensions, especially cost, could further enhance its realism and facilitate more intricate multi-objective analyses and optimizations. The vision of the authors is to continue working toward a unified approach for planning and controlling projects in their multiple managerial dimensions, which could be enabled by even higher-dimensional singularity functions (Lucko and Su 2014).

While the model was used in this study to address interior finishing activities, future studies can extend its scope to additional types of activities. In particular, it could support the planning of earthmoving activities that involve different crews and types of equipment working concurrently on a site. Further research can also target the extension of the model beyond its current three dimensions, in order to address, for example, relations between activities that are carried out at different heights on a construction site, rather than on a single horizontal plane.

Future research should also investigate which type and parameters of evolutionary algorithms perform best in efficiently identifying optimum solutions with this new model. Moreover, while the mathematical formulation of the entire model is not restricted to any level in the managerial hierarchy of task, process, operation, activity, project, or program, it could be examined how the particular position within such a Work Breakdown Structure affects the type and interactions of temporal and spatial constraints to which elements of the schedule are subjected. Furthermore, the conceptual relationship of the new – still static – temporal-spatial scheduling model on the one hand and dynamic simulations, especially discrete event simulations, on the other hand should be explored further, which may lead to synergy to further support decision-makers.
Acknowledgement

The authors gratefully acknowledge the constructive suggestions by two anonymous reviewers.

References


Table 1: Paths Coordinates

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<td>Remove F to C</td>
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<td>27.0</td>
<td>93.5</td>
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Table 2: Progress Rates

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<tr>
<th>Area or Sector</th>
<th>Total Path Length</th>
<th>Activities</th>
<th>Successor</th>
<th>Progress Rate</th>
<th>Crews</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A, 1B, 2, 3</td>
<td>1A: 48 m 1B: 48 m 2: 92 m 3: 28 m</td>
<td>Demo Fire Protection</td>
<td>Open Plenum Bolts</td>
<td>10 m/d</td>
<td>1</td>
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<tr>
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<td></td>
<td>Open Plenum Bolts</td>
<td>Demo Filters and Modules</td>
<td>12 m/d</td>
<td></td>
</tr>
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<td></td>
<td>Demo Filters and Modules</td>
<td>Demo Ducts</td>
<td>7 m/d</td>
<td></td>
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<td></td>
<td></td>
<td>Demo Ducts</td>
<td>N/A</td>
<td>6 m/d</td>
<td></td>
</tr>
<tr>
<td>From D across 1B and E</td>
<td>Original Storage (27, 14.5) to Exit (0, 67) is 60 m</td>
<td>Remove D to E</td>
<td>Sector D activities</td>
<td>Depends on number of batches and storage location</td>
<td></td>
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<tr>
<td>From E</td>
<td>New Storage (27, 50) to Exit (0, 67) is 32 m</td>
<td>Remove F to C</td>
<td>Sector F activities</td>
<td>Depends on number of batches and storage location</td>
<td></td>
</tr>
<tr>
<td>From F across C</td>
<td>Original Storage (27, 93.5) to Exit (86.5, 108) is 62 m</td>
<td>Removes F to C</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>From C</td>
<td>New Storage (58, 93.5) to Exit (86.5, 108) is 2 m</td>
<td></td>
<td></td>
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<tr>
<td>A, B, C, D, E, F</td>
<td>A: 348 m B: 252 m C: 348 m D: 540 m E: 496 m F: 522 m</td>
<td>Demo Filters</td>
<td>Demo PEDs/Metal Tiles</td>
<td>76 m/d</td>
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<tr>
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<td></td>
<td>Demo PEDs/Metal Tiles</td>
<td>Demo Stanchions</td>
<td>90 m/d</td>
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<td></td>
<td>Demo Stanchions</td>
<td>Repair Flooring</td>
<td>100 m/d</td>
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<td>Repair Flooring</td>
<td>N/A</td>
<td>180 m/d</td>
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From D across 1B and E: Sector E and D activities depends on number of batches and storage location.

From F across C: Sector F activities depends on number of batches and storage location.
<table>
<thead>
<tr>
<th>Version</th>
<th>Number of Batches [-]</th>
<th>Quantity per Batch [m]</th>
<th>Minimum Distance of Removing Modules from Sector D and Area 1B [m]</th>
<th>Total Project Duration [d]</th>
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<tr>
<td>Original</td>
<td>1</td>
<td>216</td>
<td>10.6 &gt; 8 m safety distance</td>
<td>161.3</td>
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<tr>
<td>Changes</td>
<td>2</td>
<td>108</td>
<td>10.6 &gt; 8 m safety distance</td>
<td>159.3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>72</td>
<td>8.2 &gt; 8 m safety distance</td>
<td>158.7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>54</td>
<td>7.8 &lt; 8 m safety distance (Violation!)</td>
<td>158.3</td>
</tr>
<tr>
<td>Optimal</td>
<td>3</td>
<td>72</td>
<td>8.2 &gt; 8 m safety distance</td>
<td><strong>158.7</strong>: 2.6 days reduction</td>
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<tr>
<td>Version</td>
<td>Number of Batches [-]</td>
<td>Quantity per Batch [m]</td>
<td>Duration per Batch [d]</td>
<td>Total Project Duration [d]</td>
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<td>------------------------</td>
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</tr>
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<td>Original: Temporary Storage Locations in Sector D (27,14.5) and Sector F (27,93.5)</td>
<td>1</td>
<td>216</td>
<td>4.00</td>
<td>161.3</td>
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<tr>
<td>Changes in Number of Batches and Temporary Storage Location in Sector E (27, 50) and Sector C (58, 93.5)</td>
<td>2</td>
<td>108</td>
<td>1.10</td>
<td>158.4</td>
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<td></td>
<td>3</td>
<td>72</td>
<td>0.70</td>
<td>158.0</td>
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<td>4</td>
<td>54</td>
<td>0.53</td>
<td>157.8</td>
</tr>
<tr>
<td>Optimal</td>
<td>4</td>
<td>54</td>
<td>0.53</td>
<td>157.8: 3.5 days reduction</td>
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### Table 5: Cost Data

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<th>Item</th>
<th>Basis</th>
<th>Percent of Basis</th>
<th>Amount</th>
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<td>Direct cost</td>
<td>N/A</td>
<td>N/A</td>
<td>$435,608</td>
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<tr>
<td>Indirect cost</td>
<td>Total cost</td>
<td>20%</td>
<td>$108,902</td>
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<tr>
<td>Total cost</td>
<td>N/A</td>
<td>N/A</td>
<td>$544,510</td>
</tr>
<tr>
<td>Markup rate</td>
<td>Total cost</td>
<td>25%</td>
<td>$136,128</td>
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<tr>
<td>Owner retainage</td>
<td>Total cost + markup</td>
<td>10%</td>
<td>$68,064</td>
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<td>Financing interest</td>
<td>Bank loan</td>
<td>2% per year</td>
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<td>Credit limit</td>
<td>Total cost</td>
<td>75%</td>
<td>$408,383</td>
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</table>

**Note:** Labor cost is $200 per worker-day; total cost = direct cost + indirect cost; markup rate covers overhead and profit; bill-to-payment delay = 30 days; retainage is withheld from each pay and released at the last pay time.
Table 6: Cash Flow for Different Numbers of Batches Removed with Change in Temporary Storage Locations

<table>
<thead>
<tr>
<th>Storage</th>
<th>Number of Batches</th>
<th>Total Cost</th>
<th>Profit at Finish</th>
<th>Maximum Negative Balance</th>
<th>Duration</th>
<th>Approximate Savings by Early Finish</th>
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<td>Original Location</td>
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<td>$544,510</td>
<td>$233,152</td>
<td>-$184,106</td>
<td>161.3</td>
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<td>$542,882</td>
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<td>3</td>
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<td>-$189,925</td>
<td>158.7</td>
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<td>4</td>
<td>$543,602</td>
<td>$233,148</td>
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<td>New Location</td>
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<td>$231,247</td>
<td>-$187,165</td>
<td>157.8</td>
<td>$35,000,000</td>
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Figure 1: Time-Space Chart
Figure 2: Location-Based Scheduling Diagram
(modified after Dagan and Isaac 2015)
Figure 3: Initial Configuration in 3D

(a): $x_1$-$y$ Plane

(b): $x_2$-$y$ Plane

(c): $x_1$-$x_2$ Plane
Figure 4: Consolidated Configuration in 3D

(a): $x_1$-$y$ Plane  
(b): $x_2$-$y$ Plane  
(c): $x_1$-$x_2$ Plane

Figure 4: Consolidated Configuration in 3D
Figure 5: Geometry of Activity and Buffer
Figure 6: Layout of Case Study Construction Site
Figure 7: Removal in One Batch
Figure 8: Removal in Two Batches
Figure 9: Removal in Three Batches
Figure 10: Removal in Four Batches
Figure 11: Cash Flow Diagram for One Batch
Figure and Table Captions

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