MODELING EARLY PAYMENT DISCOUNTS AND LATE PAYMENT FEES WITH SINGULARITY FUNCTIONS

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Abstract: Cash flow management is a vital concern of construction contractors. To break its vicious cycle of ‘pay as late as possible, get paid as early as possible’ in which the project participants may engage to their mutual detriment, potential incentives and disincentives that are used in financial transactions should be systematically investigated. Both are time-dependent functions that define a discount or surcharge based on whether a transaction is performed before or after a deadline. They can thus be expressed by so-called singularity functions, which are activated on said cutoff date. The new model expands prior research on cash flows by linking early (prompt) payment discounts – for which a practical nomograph is provided – directly with their counterpart of late payment fees. The values of both can be calculated from the individual financing interest of the participants to assess different scenarios based on their relative time value of money. They thus gain the ability to make financially informed decisions on offering a discount and imposing a fee appropriately, and accepting the discount or incurring a fee, respectively.

1 INTRODUCTION

Cash flow management is crucial for the success of a project, especially from the profitability perspective. Furthermore, the timely issue of cash flow is critical for all participants in the project. However, different participants’ motivations will lead to diverse behaviours. From a view of the time value of money (TVM), the payer (participant who pays bill) has the intention to pay late and less if possible, whereas the payee (participant who sends bill) wants to receive payment earlier and more if probable (Su and Lucko 2014b). To a certain extent, the payer’s intention has been realized in construction projects. For example, due to the motivation that the owner requires satisfactory performance from the contractor, retainage and bill-to-pay delay terms are ubiquitous in contractual payment requirements. As a result “the contractor tends to act as financier until the later stages of the project” (Green 1989, p. 55). Worse, pay when/if paid terms and delayed pay seriously impede payees, e.g. first and second-tier subcontractors, and may also create negative consequences for payers: Both public and private owners faced complaints from contractors and subcontractors that they did not receive the pay in a timely manner (Sweet et al. 2014). In public projects, according to the Federal Prompt Payment Act, “a contractor must pay its subcontractors for satisfactory performance within seven days of receiving payment from the federal agency. Failure to pay on time subjects the contractor to an interest fee owed to the subcontractor. Subcontractors have the same obligation to pay sub-subcontractors.” (Sweet et al. 2014, p.349). In private projects, unpaid contractors and sub-contractors may even have mechanic’s liens on properties (Construction Report 2014), which could make owed payments becoming more enforceable (Sweet et al. 2014). Fairness in both timely business transaction is important for the success of projects, which “the contractor should not be required to complete work at a loss, and the owner should not have to pay more than a reasonable amount of profit on any given item” (Gransberg and Riemer 2009, p. 1140). As a result, the fairness of the
contractual payment term should be considered from both the payer’s and payee’s perspectives. As Table 1 shows, the possible deals between payer and payee are either earlier payment with a discount or late payment with a fee, where the former term prompts the payer to pay in a less but earlier manner, while the latter requires the payer to pay more if it pays later.

Table 1: Payer and Payee Strategies (Adapted from Su and Lucko 2014b)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Timing</th>
<th>Amount</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payee</td>
<td>Earlier</td>
<td>More</td>
<td>None</td>
</tr>
<tr>
<td>Payer</td>
<td>Later</td>
<td>Less</td>
<td>None*</td>
</tr>
<tr>
<td>Payee</td>
<td>Earlier</td>
<td>Less</td>
<td>Deal (discount)</td>
</tr>
<tr>
<td>Payer</td>
<td>Later</td>
<td>More</td>
<td>Deal (fee)</td>
</tr>
</tbody>
</table>

*Result exists if considering retainage and bill-to-pay delay

2 LITERATURE REVIEW

Hill and Riener (1979) modeled a situation that a retail firm offers a discount to those customers who pay their bills earlier than others and searches its optimum discount. The model relied upon assuming that some customers wish to take a discount, while others do not. However, without a bilateral motivational explanation of the feasibility of such discount, such hypothesis of randomness does not explain why they wish to take a discount. As de la Garza and Melin (1986) noted, the construction contract should permit prepayment to mitigate inflation. Ng et al. (1999) systematically analyzed early payment discount terms across various industries, and summarized the factors ruling behind. Touran et al. (2004) researched how prompt payment provisions influence the profitability of contractors in the transportation area. Cui et al. (2010) considered early payment discount in a system dynamics model for cash flow and derived macro-level strategies, however, the micro-level was omitted, which may encumber its application in industry. Kouvelis and Zhao (2012) explored the discount term (trade credit) between supplier and retailer by using game theory. Yet the precondition – newsvendor-like retailers – is inapplicable for a construction contract, wherein the work scope is explicitly defined. Al-Hussein et al. (2013) advocated floor and ceiling discount concepts for construction transaction, but its utilization is also hindered by lacking a theoretical foundation.

Late payment fee is the counterpart to discount, which many companies do not spell out explicitly. A reason may that the company does not wish to antagonize its customers. This is similar to charging fee for overdue payment of credit card. Another analogy is charging a fee if paying tax or filing a return after the deadline: “If you pay your taxes late, the fee is usually ½ of 1% of the unpaid amount for each month or part of a month the tax is not paid... This fee is in addition to interest charges on late payments” (IRS 2013, p. 91). For construction, the law requires that a “prime contractor who violates the law (prompt pay) is subject to licensing disciplinary action and must pay the subcontractor a fee of 2 percent per month in addition to normal interest” (Sweet et al. 2014, p.349). Su and Lucko (2014b) comprehensively explored the mechanism of prompt payment discount using a synthetic cash flow model with singularity functions, but omitted the late payment fee. A gap exists in comprehensively analyzing and unifying the financial phenomena of early payment discount and late payment fee. For this three Research Objectives are set: 1. Expand the synthetic cash flow model to suit both the early payment discount and late payment fee; 2. Explore mathematically the range of conditions for different values of the feasible late payment fee; 3. Chart graphically in form of nomographs these feasible ranges of conditions for the late payment fee.

3 MODEL CASH FLOW WITH SINGULARITY FUNCTIONS

A central issue in defining early payment discount and late payment fee is fairness for both sides, which “the payee wants to offer and the payer wants to take” (Su and Lucko 2014b, p. 8). It requires a method that can effectively and efficiently calculate the net cash flow for both payer and payee under different discount or fee rates on early or late pay periods. Previous methods fall short of balancing between them: A chronological approach calculated the balance of cash flow at the end of each period by treating actual
simultaneous transactions (charging interest and receiving pay) sequentially by inserting an infinitesimal time $\varepsilon$ (Elazouni and Metwally 2005; Halpin and Woodhead 1998). This is accurate, but lacks efficiency, because it is not formularized. As a result, calculating 'what if' scenarios with this chronological method has to arduously repeat the process. Even though it can be automated with computers, vital relations among those variables cannot be easily explored, especially to identify dominant effects. On the other hand, previous studies focused on computational efficiency of numerous types of cash flow models, e.g. linear programming (Yang et al. 1993) or heuristics (Alghazi et al. 2012; Neumann and Zimmermann 2000). Their problem was opposite to the chronological method; omitting or simplifying central details of cash flow to make the model simple enough to be implemented with a computationally efficient algorithm. For example, financing interest, retainage, and periodical phenomena were omitted from some models, which impeded the reliability and realism of their output. An ideal cash flow model should reflect essential characteristics and also provide an advantage in terms of computational efficiency, following the moniker ‘as simple as possible, as complicated as necessary’ as advocated by Ockham’s razor (c. 1287-1347).

### 3.1 Singularity Functions

Each basic term within singularity functions per Equation [1] is symbolized distinctly by pointed brackets. It performs one case distinction by evaluating the current value of the independent variable $y$ (here time, for consistency with previous research) as to whether it is smaller than the cutoff value $a$ or not. If so, it remains at zero, otherwise it is activated by treating the pointed brackets as round brackets of traditional algebra. The independent variable $z(y)$ here is cost. The exponent $n$ determines the behavior (i.e. shape) of the curve once active; low orders often suffice in models, e.g. $n = 0$ for a step or $n = 1$ for a slope. The factor $s$ then determines the intensity (i.e. strength) and takes its exact meaning from said behavior. Equation [1] can be integrated and differentiated in analogy to traditional calculus. A complete singularity function is the summation of basic terms per Equation [2], where $i$ is a running index within their count $m$.

\[
[1] \quad z(y)_{\text{basic}} = s \cdot (y-a)^n = \begin{cases} 0 & \text{for } y < a \\ s \cdot (y-a)^n & \text{for } y \geq a \end{cases}
\]

\[
[2] \quad z(y)_{\text{sing-func}} = \sum_{i=1}^{m} s_i \cdot (y-a_i)^n
\]

### 3.2 Synthetic Cash Flow Model

The arduousness of a chronological approach reduces its calculating efficiency. Yet calculating the TVM of cash flow is very important for accuracy. A particular phenomenon, periodicity, can be exploited to aid in the formulization: Since progress pay and month-end interest are initiated periodically, then the timing of month-end balances with TVM can be modeled if such repeatable cash flows are modeled by defining a ‘signal’ function. Another concept of viewing a balance, the ‘investment pool’ of engineering economics, is also helpful for the formalization, whereby the balance with TVM is equal to the difference of the future values of cost and of pay at the same time (Park 2011). This new cash flow model was called a ‘synthetic cash flow model’, to distinguish it from traditional chronological balance calculation (Su and Lucko 2014a). The synthetic cash flow model is a group of functions that formalizes variables of cash flow as parameters in equations. Figure 1 shows the steps of the synthetic cash flow model, where general input parameters at the activity level are: $z$ is the dependent variable of cost, total cost $C$, markup $M$, retainage $r$, monthly interest $i$, $y$ is the independent variable of time, duration $D$, shift $d$, and delay $d_2$, bill period $p$, bill-to-pay delay $b$, planned start $a_S$ and finish $a_F$, where $a_i = a_s + d_i$, $a_F = a_f + d_1 + d_2$ (Su and Lucko 2014a).
3.2.1 Signal Functions

Signal functions can control a periodic phenomenon like receiving progress pay and charging month-end interest. They are generated by introducing round down \( \lfloor \cdot \rfloor \) and up \( \lceil \cdot \rceil \) operators into the terms with the independent variable \( y \). They work as follows: The term \( \lfloor y \rfloor - a \) is a step function that turns on when \( y \) equals \( a \), while the other term \( \lceil y \rceil - (a + 1) \) is also a step function, but turns on when \( y \) is just larger than \( a \). Their difference returns a periodic signal. Figure 2 shows the case when the cutoff \( a \) is 0: Here \( \lfloor y \rfloor - 0 \) has right-continuous jump discontinuities per Figure 2(a) as represented by solid and hollow circles, whereas \( \lceil y \rceil - 1 \) has left-continuous jump discontinuities per Figure 2(b). Figure 2(c) is the profile of subtracting (b) from (a), which gives a periodic. Its period, amplitude, and start and finish can be controlled with additional parameters (Su and Lucko 2014a, Su and Lucko 2013). To apply it to a pay signal and charging interest signal per Equation [7] and [8], the bill period \( p \) affects the cycle time of the pay signal, it is applied as a divisor (since interest is monthly, the divisor 1 is omitted in Equation [8]). A virtue of signal functions is that it allows the cutoff \( a \) to be any fractional number. Thus if an activity start or finish \( a^* \) and \( a_F^* \) are fractions of periods due to shifts and delays, Equations [7] and [8] can model fractional signals at period boundaries. This allows potential integration of signals with schedule research.

\[
\sum_{\text{pay_signal}}(y) = \left[ \left( \frac{y-b}{p} \right) - \left( \frac{a^*}{p} \right) \right] - \left[ \left( \frac{y-b}{p} \right) - \left( \frac{a^*_F}{p} \right) \right] + \left[ \left( \frac{y-b}{p} \right) - \left( \frac{a^*}{p} + 1 \right) \right] - \left[ \left( \frac{y-b}{p} \right) - \left( \frac{a^*_F + 1}{p} \right) \right]
\]

Figure 1: Steps of Synthetic Cash Flow Model with Singularity Functions

Figure 2: Mechanism of Signal Function
3.2.2 Pay Functions

Since each signal is a value between 0 and 1, multiplying it with the intensity factor performs a periodic sampling. Defining a cost intensity factor $C \cdot (1 + M) / (D + d_2)$ returns each pay per Equation [9]. Note $p$ is for both integer and non-integer bill period cases. To subtract retainage, Equation [10] calculates the retained amount at each pay time and Equation [11] is the pay with retainage, releasing the accumulated sum of the retained amount when the project is finished. Treating the result from Equation [11] as the principal, Equation [12] calculates the future value of pay at any time during the schedule.

$$z_{\text{each\_pay}}(y) = \frac{C \cdot (1 + M)}{D + d_2} \cdot p \cdot z_{\text{pay\_signal}}$$

$$z_{\text{ret}}(y) = r \cdot z_{\text{each\_pay}}(y)$$

$$z_{\text{pay\_less\_ret}}(y) = z_{\text{each\_pay}}(y) - z_{\text{ret}}(y) + \left( \frac{y - b}{p} - \frac{a_F^{\ast}_{\text{project}}}{p} \right)^0 \cdot z_{\text{pay\_signal}} \cdot \sum z_{\text{ret}}(y)$$

$$z_{\text{FV\_pay}}(y) = z_{\text{pay\_less\_ret}} \left( (a_s^{\ast} + b + 1) \cdot (1 + i) \cdot (\lfloor y \rfloor - \lfloor a_s^{\ast} + b + 1 \rfloor)^i \cdot (y - \lfloor a_s^{\ast} + b + 1 \rfloor)^0 \right) + z_{\text{pay\_less\_ret}} \left( (a_F^{\ast} + b + 2) \cdot \frac{1}{i} \left[ (1 + i) \cdot (\lfloor y \rfloor - \lfloor a_F^{\ast} + b + 1 \rfloor)^i - (1 + i) \cdot (\lfloor y \rfloor - \lfloor a_F^{\ast} + b + 1 \rfloor)^0 \right] \right) + z_{\text{pay\_less\_ret}} \left( (a_F^{\ast} + b) \cdot (1 + i)^i \cdot (y - \lfloor a_F^{\ast} + b \rfloor)^0 \right)$$

3.2.3 Cost with Interest Functions

Analogous to pay, applying an exact interest formula (Lucko 2013), the future value of cost is calculated by introducing a charge interest signal into the exponent terms per Equation [13]. Different from Equation [9], it is multiplied with an intensity factor $C / (D + d_2)$, because cost does not include markup or retainage.

$$z_{\text{FV\_cost}}(y) = \frac{C}{D + d_2} \cdot \frac{1}{\ln(1 + i)} \cdot \left[ (1 + i)^{z_{\text{int\_signal}}} - 1 \right] \cdot (1 + i)^{(\lfloor y \rfloor - \lfloor a_s^{\ast} + 1 \rfloor)^i} \cdot \left( y - \lfloor a_s^{\ast} + 1 \rfloor \right)^0 + \left[ (1 + i)^{(\lfloor y \rfloor - \lfloor a_s^{\ast} + 1 \rfloor)^i} - (1 + i)^{(\lfloor y \rfloor - \lfloor a_s^{\ast} + 1 \rfloor)^0} \right] + (1 + i)^{(\lfloor y \rfloor - \lfloor a_s^{\ast} \rfloor)^i} \cdot \left( y - \lfloor a_s^{\ast} \rfloor \right)^0$$

3.2.4 Balance Functions

After obtaining both the future values of pay and cost, and applying the aforementioned ‘investment pool’ concept, Equations [14] and [15] give the balance with TVM, which is the last step in Figure 1. Note that Equations [7] through [16] compose the synthetic cash flow model, whose steps have been explained. Note that all individual equations that are explained for this synthetic cash flow model can be inserted into an overall equation to ultimately return a single general balance function, which is omitted here for brevity.

$$z_{\text{step\_cost}}(y) = \frac{C}{D + d_2} \cdot \left[ (\lfloor y \rfloor - a_s^{\ast})^i - (\lfloor y \rfloor - a_F^{\ast})^i \right]$$

$$z_{\text{int\_at\_signal}}(y) = z_{\text{cost\_int}}(y) - z_{\text{step\_cost}}(y)$$
4 EARLY PAYMENT DISCOUNT

An early payment discount is the first deal between payer and payee in Table 1. Its feasibility has been discussed in previous research by the authors (Su and Lucko 2014b), as is summarized in the following.

4.1 Floor Discount

The lower limit of the early payment discount is called the floor discount, which would be affected by the payer’s interest rate. No matter how generous the discount is, this is the minimum that is has to be so that the payer wants to take it (otherwise no deal will be possible). Note the new parameters in the synthetic model, where \( l \) is the early pay period and \( \rho \) is the discount rate (e.g. 2 / 10, net 30, \( l = 10 \text{ days} \) and \( \rho = 2\% \text{ base on bill} \)). Here ‘\( \rho / l \), net \( b \)’ means that the payer can pay the bill less a discount \( \rho \) if paid within \( l \) days, or pay the full amount within \( b \) days. The pay signal shifts leftward by \( b - l \) days. Its cost intensity factor must be \( C \cdot (1 + M \cdot (1 - \rho)) / (D + d_2) \). As a result, it would use the payer’s interest rate to calculate the future value of pay for both early and normal pay scenarios, to let the former be smaller than the latter per Equation 17. Only in this case it is favourable for the payer to pay earlier but less from a TVM view. The floor discount is also the payer’s indifference discount, because taking it or not has the same effect.

\[
4.1 \quad \begin{align*}
    z_{_F V \_pay \_disc \_pay} (a_F^* + b) & \leq z_{_F V \_pay \_balance} (a_F^* + b) \\
    \rho_{floor} & = 1 - (1 + i_{_p ay er})^{l-b}
\end{align*}
\]

4.2 Ceiling Discount

Similar to the floor, the ceiling discount is the upper limit of the early payment discount, which is defined by the payee’s interest rate. Because no matter how urgently the payer wants to accept a discount, if the payee does not want to offer it, it is still infeasible. In this case, one must compare the balance function of the payee between early and normal pay scenarios, to let the former be larger than the latter per Equation [18]. It models that the payee earns more if the payer pays earlier. Since cost is not affected by taking the discount or not, Equation [18] is further simplified to comparing future pay functions for both scenarios. As a result, the ceiling discount has the same pattern as the floor discount. It is also the payee’s indifference discount, because it has the identical effect for the payee, regardless whether the payer takes it or not.

\[
4.2 \quad \begin{align*}
    z_{_balance \_disc} (a_F^* + b) & \geq z_{_balance} (a_F^* + b) \\
    \rho_{ceiling} & = 1 - (1 + i_{_p ayee})^{l-b}
\end{align*}
\]

4.3 Feasible Early Payment Discount Term

The condition for the feasibility of the early payment discount is that the floor discount should be lower than the ceiling discount per Equation [19]. Substituting the results from Equation [17] and [18] into this condition returns a simple result: As long as the payer’s interest is lower than the payee’s interest, there exists a feasible range for the discount. Note that not only the feasible discount range can be calculated from the equation; the feasible early pay period \( l \) can also be computed. For example, assume the payer’s interest is 4%, payee’s 8%, and the payee wants to give a ‘\( \rho / l \), net 30’ discount. Per Equation [20], the feasible discount range is 2.5808% to 5.0013%. If the payee wants to offer ‘2 / l, net 30’, the feasible early period per Equation [21] is any integer time between 15 and 22 days. Users who know two early payment discount variables for their particular case can thus directly calculate the third value using these formulas.

\[
4.3 \quad \begin{align*}
    \rho_{floor} & \leq \rho_{ceiling} \\
    i_{_p ayer} & \leq i_{_p ayee} \\
    \rho & \in \{ \rho_{floor}, \rho_{ceiling} \} \\
        & \{ 2.5808\%, 5.0013\% \} \\
    l & \in \left\{ b + \log_{1+i_{_p ayer}} (1-2\%), b + \log_{1+i_{_p ayee}} (1-2\%) \right\} \\
        & \{ 14.5469, 22.1248 \} \text{days}
\end{align*}
\]
5 LATE PAYMENT FEES

A late payment fee is the second deal between payer and payee in Table 1. This lets the payer pay later but more. From the view of construction law, the term ‘fee’ paid from the owner to the contractor is similar to ‘liquidated damage’, by which the contractor compensates the owner for delaying the project finish time.

5.1 Floor of Late Payment Fee

Note the new parameters in the synthetic model, where \( l \) is the period between the time of normal pay and late pay and \( i' \) is the fee (e.g. \( b = 30 \) days, if pay is overdue, charging \( i' = 2\% \) monthly interest). From the payee’s view, the balance of the delay situation should be larger or at least equal to the normal case per Equation [22]. Solving it returns an apparent result that the late payment fee should be at least equal to the payee’s financing interest, which serves as the floor value of the fee. Moreover, from the payer’s view, a fee actually could have two effects for the payer: It is favorable for the payer to pay late rather than on time per Equation [23]. Or it is favorable for the payer to pay on time per Equation [24]. Obviously, from the payee’s view, wanting the fee can lead the payer to select the second case. Two scenarios of inequalities for \( i_{\text{payee}} \) and \( i_{\text{payer}} \) exist (assuming that the payer borrows to pay): If \( i_{\text{payee}} < i_{\text{payer}} \), then the payee requires a late payment fee between the range of \( \{i_{\text{payee}}, i_{\text{payer}}\} \), and the payer will pay late, because the fee is smaller than their own loan interest. Here the floor fee should be at least larger than \( i_{\text{payer}} \). Otherwise if \( i_{\text{payee}} > i_{\text{payer}} \), then the floor fee is applied. The floor fee is the maximum of \( i_{\text{payee}} \) and \( i_{\text{payer}} \) per Equation [25].

\[
\begin{align*}
z_{\text{balance-delay}} \left([a_F^*] + b + l\right) &\geq z_{\text{balance}} \left([a_F^*] + b + l\right) \quad \Rightarrow \quad C \cdot \left(1 + M \right) \cdot \left(1 + i'\right) \cdot \left(1 + i_{\text{payee}}\right)^{(l - t)} \nonumber \\
&\geq \frac{C \cdot \left(1 + M \right) \cdot \left(1 + i_{\text{payee}}\right)}{D + d_2} \cdot \left(1 + i'\right) \cdot \left(1 + i_{\text{payee}}\right)^{(l - t)} \nonumber \\
&\Rightarrow \quad i' \geq i_{\text{payee}} \nonumber \\
\end{align*}
\]

[22]

\[
\begin{align*}
z_{\text{pay-delay}} \left([a_F^*] + b + l\right) &\leq z_{\text{pay}} \left([a_F^*] + b + l\right) \quad \Rightarrow \quad C \cdot \left(1 + M \right) \cdot \left(1 - r\right) \cdot \left(1 + i_{\text{payer}}\right)^{(l - t)} \nonumber \\
&\leq \frac{C \cdot \left(1 + M \right) \cdot \left(1 - r\right)}{D + d_2} \cdot \left(1 + i_{\text{payer}}\right)^{(l - t)} \nonumber \\
&\Rightarrow \quad i' \leq i_{\text{payer}} \nonumber \\
\end{align*}
\]

[23]

\[
\begin{align*}
z_{\text{pay-delay}} \left([a_F^*] + b + l\right) &\geq z_{\text{pay}} \left([a_F^*] + b + l\right) \quad \Rightarrow \quad i' \geq i_{\text{payer}} \nonumber \\
\end{align*}
\]

[24]

\[
\begin{align*}
i_{\text{floor}} &= \max \{i_{\text{payee}}, i_{\text{payer}}\} \nonumber \\
\end{align*}
\]

[25]

5.2 Ceiling of Late Payment Fee

A ceiling of the late payment fee must exist to guarantee fairness for both sides and because a mere punishment is not allowed by courts. However, different from the early pay case, from both payee’s and payer’s views per Equations [22] (payee’s balance of late pay should be larger than the balance of the normal pay case) and [24] (payer’s late pay should be larger than in the normal pay case), both conclude the same direction of the inequity (\( \geq \)). As a result, both \( i_{\text{payee}} \) and \( i_{\text{payee}} \) define \( i_{\text{floor}} \). This lacks the condition to define \( i_{\text{ceiling}} \). An open-ended late payment fee will increasingly resemble a punishment, not liquidated damages. A possible solution for this is to set a ceiling fee by negotiations between payer and payee.

5.3 Nomograph for Both Discount and Fee

An early payment discount per Equations [17] and [18] has the variables \( i, l, \) and \( b \), which can be plotted as nomographs to show their relationships among each other. Since \( i, l, \) and \( b \) have two types of units (% from \( i \) and time units from \( l \) and \( b \)), two nomographs can be plotted: Interest-discount and time-discount. In the interest-discount nomograph per Figure 3, each line shows how discount changes with growing
interest for a specific early pay period case. Since $i$ is the independent variable and exponent $l - b$ is the constant of each line; the profile of each line is essentially a power function. It monotonically increases, because the derivative of such power function is positive. To use this nomograph, first select the line for the planned $l$ value. Second, find the payer’s $i$ (4%) and payee’s $i$ (8%) on the horizontal axis. Third, find the cross on the $l = 10$ profile. Finally, draw a horizontal line crossing the vertical axis to find the floor and ceiling discounts. For the late payment fee, since $i_{\text{payee}}$ is larger than $i_{\text{payer}}$, so the $i_{\text{floor}}$ should equal $i_{\text{payee}}$.

The time-discount nomograph per Figure 4 switches the horizontal axis with time. Each profile represents how discount changes with growing early pay period for a specific interest case. Since the variable $l - b$ in the exponent is the independent variable and the base $1 + i$ is a constant, each line will be essentially an exponential function. It monotonically decreases, because its first order derivative negative.

6 CONCLUSIONS

On-time pay is a serious issue for both payee and payer, which directly affects the economic success of the former and keeps project running smoothly for the latter. This paper started at two possible deals that could be made between payee and payer in the payment terms: Early payment discount defines a case that lets the payer pay promptly (before the due date) less a discount, and late payment fee regulates the opposite case that add an extra charge to the pay if the payer pay the bill late. To analyze such situations, it has established a cash flow model that can model them effectively and efficiently. A synthetic cash flow model was presented as a group of equations with singularity functions, which was explored in previous research by the authors. Then conditions for the feasible early payment discount and late payment fee have been calculated from the cash flow model. They can also be plotted in nomographs, which provide a deeper understanding for such payment terms and helps project participants to make rational decisions.

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