Toward Mitigating Risk in Network Schedules
by Valuating Distributed Project Float for Trade
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Abstract
This paper proposes a new approach toward mitigating risk in construction projects that is based on the concept of float as an essential and inherently valuable resource. Whereas risk occurs in different forms for many reasons, it is always measurable by its (negative) impact on the overall performance of the project, most importantly in terms of time and/or cost. This new approach therefore focuses on float as temporal flexibility at the activity level that can reduce such impact on the activity. Previous studies have suggested that float should be used for the benefit of the overall project rather than only one participant. Moreover, various suggestions were made that it should be allocated among various participants in proportion to a measure of their relative importance. Eventually, the goal of such float allocation is to make it freely tradable among the participants, as posited by a study on ‘float as a commodity’. This paper therefore first examines float types and distills a fundamental paradox that exists in defining float versus criticality in network schedules. It then describes how a barely explored type, project float, which occurs after the calculated end date and lasts until the contractually specified deadline, can be used to mitigate risk. The question of valuation, once it has been allocated with a suitably distributed ownership, is addressed by treating its potential use or trade as an option in the decision-making by project managers. Basic tenets of real options are described. A worked schedule example shows how such valuation of distributed project float can be performed conceptually.

Introduction
Planning and controlling construction projects, which consist of multiple activities that will be performed by various participants in an intricate sequence with branching and merging dependencies, must be ever mindful of the constant presence of risk. Risk management is therefore an important key component within construction management. Many previous studies have identified sources of risk and ranked factors that contribute to its presence. Such origins of risk lie in the physical nature of construction work, which produces large facilities by assembling materials and systems on sites that are affected by climate, weather, and geological conditions, as well as in the business environment that surrounds them. Financial, legal, and labor considerations play important roles that can strongly impact the success or failure of a project, even if all physical operations are performed flawlessly. All of them bear different risks that merit a closer analysis. A fundamental goal of construction management is therefore to identify and mitigate risks as best as possible. Reducing inherent project risk will reduce costs, shorten durations, and improve quality and safety.

Risk in Construction Projects
Risk can be defined as a multiplicative combination of the probability of a negative event occurring with the severity of its impact should it actually occur. In other words, high risk can stem either from a high probability, a very negative undesirable outcome, or both. By splitting the concept of risk into these two components it becomes easier to model and analyze it for the purpose of mitigation. Estimating uncertainty by itself is a research area, where studies have attempted to e.g. extract probabilistic distributions from real-world data, create models that incorporate uncertainty in various project processes, and assess its impacts on project performance (AbouRizk et al. 1994, Vaziri et al. 2007, Trofin 2004). Establishing a valid numerical value for uncertainty is beyond the scope of this paper. It is assumed that such measure can be calculated from historical data or estimated from the project manager’s expertise.

Types of Risks
Construction projects are exposed to manifold risks, which can be classified according to the nature of their origin, e.g. including environmental, financial, other business, and political risks (Klemetti 2006). The categories may be expanded further into subtypes until eventually arriving at individual factors, such as e.g. the existence of unforeseen subsurface conditions on the construction site and an occurrence of extreme temperatures or significant precipitation as environmental risks, the unpredictability of timely payments that enable a viable cash flow, as well as a non-zero potential for bankruptcy of any project participant as financial risks, a plethora of issues such as e.g. variations in
raw materials prices, an increased competition in the local market, or a weakened national or even global economy, as business risks, and adverse legislation, reduced tax incentives, or internal conflicts as examples of political risks.

**Impacts of Risks**

As explained in the previous section, the extent and composition of risk will be completely unique to each project and indeed to each activity therein. Regardless of the bewildering variety of different types of risk, it is surmised that their impacts are much clearer in nature and much easier measured. Impacts of interest in this analysis either directly result in increased costs to the participants that are not reimbursed by the owner under e.g. a change order, or will result in a delayed schedule that causes extra costs, if acceleration remains possible, or leads to monetary penalties. This paper focuses on the negative impact of risks on network schedules, which are used ubiquitously to analyze projects (Galloway 2006). This impact will materialize as time delays, which can be categorized into two types:

- Activities start late due to predecessors, but perform at the planned productivity and have no changed duration;
- Activities start on time, but perform at a lower-than-planned productivity and thus also have a longer duration.

Combinations of these two delay types can often occur. In the worst case this phenomenon can create a cascading ‘ripple effect’ that may aggravate throughout the network schedule along its sequence. However, both of these delay types are effectively mitigated by consuming float that is located anywhere after the activity finish. It is therefore necessary to examine float, which is an essential and inherently valuable resource and provides temporal flexibility. Acceleration measures that increase the productivity of activities are less desirable than using float for minimizing a delay that has already occurred, because they are more costly due to using more or better resources, i.e. hiring extra or higher skilled labor, renting extra or more powerful equipment, or other innovative technology. Moreover, these costs will typically increase significantly for decreasing the activity durations, which requires a sophisticated time-cost tradeoff analysis (Liu et al. 1995). Float, on the other hand, advantageously already exists within the projects.

**Float in Construction Projects**

Float in its most basic form is the opposite of criticality. Both are determined through the aptly named critical path method (Kelley and Walker 1959). This straightforward algorithm consists of three steps; the first one – **forward pass** – begins at the overall start activity and adds the duration of all successors while following their sequence. Within each activity, its start date plus its duration equals its finish date. Whenever the current activity has several predecessors, the maximum of their finish dates is used as the new start date. Thus all of the constraints that are expressed by links in the network schedule are obeyed. In short, the forward pass calculates all dates while assuming the best case scenario that all activities actually start as early as it is possible from their sequential dependencies.

The second step – **backward pass** – performs the inverse calculation that is inverted in all of its aspects, i.e. it begins at the overall finish activity and sequentially subtracts the durations of all predecessors. Whenever said activity has several successors, the minimum of their start dates is used as the new finish date. The finish date minus the duration of each activity equals its start date. Forward and backward labels refer to the calculation being performed in the positive or negative direction along the sequence on the time axis. This backward pass uses the opposite assumption in mirror-wise analogy to the forward pass, the worst case scenario that all activities finish as late as it is possible. The third step calculates the various float types as described in the following section by comparing these results.

A major paradox can be identified within the definition of float itself. Visualizing it on a horizontal axis of temporal flexibility, all critical activities whose flexibility is exactly zero (or near zero, if a construction company may define anything less than a fixed small amount of float as ‘critical’) will be located at or near the origin. All other activities to the right of it have positive float. Float within a network schedule thus theoretically can have a maximum value of the planned project duration minus one time unit, for an imaginary activity that is one workday long and occurs on a path without any predecessors or successors where it has almost unbounded flexibility. However, defining criticality as the absence of flexibility, or float, creates two sets; one critical group of activities that is located along the longest path and all others that are non-critical, have float, and are located on secondary paths in the network schedule. Yet critical activities are exactly those that are most in need of float (Thompson and Lucko 2011). This paradox leads to the conclusion that it may be more beneficial to attempt to allocate at least some float to all activities in a project.

**Float Types**

Among the most well-known float types is **total float** (TF), which is calculated for each individual activity either as the difference between its earliest and latest start dates (start float) or between its earliest and latest finish dates...
Ownership Approaches

Ownership approaches as published in the literature have traditionally focused solely on float within network schedules, not beyond them – after the network – where CF can be found. Nonetheless, they are reviewed here to identify a feasible and acceptable way of allocation. Arguments for ownership can be categorized into three types:

- Ownership of float should predominantly or exclusively be given to the owner who commissions the project, holds the ultimate legal authority on approvals, and obviously also pays for it in its entirety (Al-Gahtani 2006);
- Ownership of float should predominantly or exclusively be given to the general contractor who “is responsible for the means, methods and techniques of construction” that include the schedule details (Pasiphol 1994, p. 3);
- Ownership of float should be shared fairly and equitably between owner and general contractor, e.g. in an even 50%-50% split (Prateapusanond 2003) to benefit the overall project (O’Brien and Plotnick 2006, p. 593): “Float time is not for the exclusive use or benefit of either the Contractor or the Owner.” However, unless an explicit pre-allocation is contractually recorded, problems may still occur due to unclear ownership and a ‘first-come-first-serve’ mentality could arise that may harm the project performance and eventually lead to legal claims.

Most approaches do not explicitly mention the equally important relationship between the general contractor and its subcontractors, but some implicitly appear to assume that activities are performed by different subcontractors. It was suggested that TF should be allocated to non-critical activities in proportion to their duration (Ponce de Leon 1982), an approach that recognizes that longer activities may incur more or longer delays, i.e. have higher risk. Obviously, the nature of the work itself, e.g. the more risky excavation versus the less risky painting, will also influence the risk. A comprehensive approach to ownership of float should therefore be mindful of the different type, size, and nature of the contribution that each participant makes to the entire project. It should comprise an algorithm that provides a clear and unambiguous allocation based on a measure of risk, using heuristics if necessary as tiebreakers. It should also be judged as fair and equitable by neutral criteria. Initial work by Thompson and Lucko (2011) has proposed an analytical model that uses an analogy of decision-making among voters of different weights to determine potential influence (akin to risk) of each critical activity within a network schedule and allocate float in proportion. Further research is needed to create and quantify a composite measure of risk that construction managers can use in practice to adequately reflect the relative contribution of each activity in terms of time and cost as well as its unique nature. This paper will assume for simplicity that the CF is allocated as DF in proportion to the duration of each activity. In other words, it assumes that duration that each subcontractor spends on the site is proportional to the individual risk that needs to be mitigated by providing DF. Note that ownership of DF does not mean that said float will actually be
relocated directly after each critical activity, as this would change all start and finish dates as calculated with the critical path method. Rather, CF remains one block after the network to be consumed by its partial owners if needed. For clarity, all participants within the network schedule itself are termed ‘subcontractors’ in this paper to emphasize that they are competitors as well as contributors, albeit of potentially different influence on the project performance. Naturally, they are still supervised by a general contractor who is ultimately legally responsible for the entire project to its owner. Assuming that none of the actual work is self-performed by the general contractor is useful, because it reduces the problem to a single type of participants for the purpose of performing this proof-of-concept analysis.

**Proposed Float Valuation**

A two-decade-old seminal paper has called for treating float as a commodity that ultimately should become tradable among project participants (de la Garza et al. 1991). While that publication had limited its view to only TF within the network schedule that was supposed to be traded among non-critical participants, not CF, and thus ignored the urgent need all critical activities to receive flexibility that mitigates risk, it nonetheless provided important impulses. It concluded “that early float would be more expensive because of the downstream impacts on the activities sharing the same path… As the project progresses, more is known about it, thus reducing the level of uncertainty and the value of total float” (ibid., p. 720). It was suggested to price TF of each non-critical activity as the difference of its latest and earliest finish cost, divided by its TF (ibid.). However, this notion that higher risk results in a higher price was not modeled explicitly. Moreover, critical activities, which have been identified as having the most urgent need of float, cannot be treated with such approach as their TF = 0. Rather, their CF should be newly examined in detail. Furthermore, such approach must also consider additional phenomena of supply and demand throughout the project. Demand for float may increase toward the later end of the project in a mounting urgency to complete it on time. For TF specifically, plenty of float will be available early during its execution, but it is an expiring resource that will decrease until it reaches zero at the calculated finish. On the other hand, CF is allocated as DF and does not expire unless it is actually consumed. It could be purchased by any critical subcontractor that still must complete their work. Their number, i.e. the size of the market wherein a trade may occur, decreases until only the last two critical subcontractors remain. A strong need to explore and formalize how to valuate such tradable float continues to exist.

**Rounding Issues**

As project float is typically a very limited resource that is much shorter than the project duration itself, with multiple critical subcontractors competing for it, the issue of dividing it becomes subject to rounding issues. More precisely, it will need to be divided into even multiples of the chosen time unit, which in construction scheduling commonly is workdays. Only is a very unlikely case are calculated allocation percentages evenly divisible among the different sized subcontractors. In most cases it will therefore be necessary to round the percentages toward integer multiples of the time unit. Rounding can use three approaches of always rounding down, always rounding up, or accountant-style rounding, which rounds down for all decimals under 0.5 and up for all decimals equal to or greater than 0.5. Allocation percentages must be converted into workdays of DF unless a more detailed time unit such as workhours is officially used. Such rounding must fulfill the condition that the sum of DF is equal to the total CF itself, as no further float is available without exceeding the contract deadline and incurring penalties. In other words, it requires scaling. In fact, it is the same issue that can be found in political voting when allocating seats of parliament among multiple parties of different sizes. Various methods were proposed historically, including those by Huntington-Hill, which is currently used for the U.S. House of Representatives (Balinski and Young 1989), Webster (also named after Sainte-Laguë), and D’Hondt (also named after Jefferson) (Toplak 2009), which have in common that they carefully scale to the desired total while only either rounding down, up, or both during their seat allocation process. In the case of allocating a small amount of CF among subcontractors of significantly different sizes, rounding down or accountant rounding may even cause individual subcontractors of small size to receive zero distributed float. In the case of the U.S. House of Representatives, each state will automatically receive at least one seat, all others are distributed proportionally. An analogous approach could be taken in construction management if a sufficiently large amount of CF exists relative to the number of critical subcontractors to avoid that any of them receives zero DF. These two potentially contentious issues of how to avoid disadvantaging small subcontractors and which rounding method would be best for scaling to the correct total CF ultimately cannot be solved mathematically, but should be addressed through negotiation and documented through appropriate contract language regarding float ownership in an extension of a previous study on contract clauses to allocate TF in network schedules (Prateapusanond 2003).
Financial Valuation Model

Once float ownership has been allocated among the critical subcontractors, the DF is available for potential use. This can take several forms, which depends on the performance of the subcontractor itself, as well as on the supply and demand for DF elsewhere in the project. The following scenarios are possible, most of which will lead to a trade:

- The subcontractor consumes none of its DF, because its activity was finished on time, and can thus trade its DF;
- The subcontractor consumes part of its DF to mitigate a delay, but still has some DF that it can trade to others;
- The subcontractor consumes all of its DF, if partial mitigation only must either still acquire additional DF from others, must use acceleration techniques at an increased cost, or must accept any potential financial penalties.

All of these scenarios can be treated as a decision-making process that uses flexibility to hedge against risk, or more specifically its negative impacts. In each case float (DF) provides this essential flexibility in the network schedule. Moreover, the subcontractor has to weigh among two or more options that connect its actual and projected schedule performance – measured in time units – with their respective financial value – measured in monetary units. In other words, float should be treated as an asset – an object of value – and accordingly should undergo a careful valuation.

Decisions on trading assets are formally addressed by option theory, which is famously defined as “a security giving the right to buy or sell an asset, subject to certain conditions, within a specific period of time” (Black and Scholes 1973, p. 637). Options can be made on physical or financial assets, e.g. agricultural, mineral, or petrochemical bulk goods that are traded in global markets, stocks, bonds, or other financial instruments. As their name implies, options are a form of contract that never carries an obligation to an action by the owner. They merely, but importantly, pay for the opportunity for their owner to act and bind a second party to guarantee accepting such decision, i.e. purchase an asset at a fixed price with a call option or sell it at a fixed price with a put option (or simply forego them if either the market price is below the call option price, or vice versa if the market price is above the put option price).

The question then becomes how to determine the value of the option itself from factors including the fixed price that it specifies, the time period over which it is valid, and the uncertainty within the underlying market. Real options are a special type that applies to making business-related decisions, which are broader in nature and are categorized into four types: Giving the right to begin a capital investment, i.e. start a project or task; to end said investment, i.e. finish a project or task; increase its size or intensity; or decrease its size or intensity (Chambers 2007). Especially the first two definitions can be newly applied to network schedules. Float (DF) should thus be valued analogously by using one of the several existing scientific approaches. Arguably the most famous one is the Nobel Prize winning Black-Scholes model (1973) that employed a partial differential equation to express the behavior of the option value. It explains that longer time periods yield a higher value (i.e. providing more flexibility to postpone making a decision). More extreme differences between fixed price and market price will also lead to a higher value. It assumed a known interest rate r to assess its Time Value of Money; a constant standard deviation for fluctuations in market value; and, for simplicity, that the option could only once be exercised, exactly at the end of its validity. Equation 1 describes a call option (purchase asset) in its famous, if complex, original mathematical form (Black and Scholes 1973, p. 644). The first term in Equation 1 is the expected value of the asset and the second term consists of the present value of the cost of investment multiplied with the risk-neutral probability. For brevity the equivalent put option (sell asset) is omitted here, as it is much less likely that a subcontractor may seek an option that guarantees being able to sell DF.

Equations 2 and 3 that are derived from evaluating the distribution, where the variance \( \sigma^2 \) is a constant standard deviation for fluctuations in market value; and, the expected value of the asset and the second term consists of the present value of the cost of investment multiplied with the risk-neutral probability. For brevity the equivalent put option (sell asset) is omitted here, as it is much less likely that a subcontractor may seek an option that guarantees being able to sell DF.

Adoption to Project Management

Exhibit 2 shows the network schedule for this example including the earliest start (ES), earliest finish (EF), latest start (LS), latest finish (LF), as well as TF and FF for each individual activity. The critical path is marked with thick arrows and activity boxes. Additionally, CF is shown after the network and its allocated form DF is shown in dashed boxes after each activity. The assumed proportionality with their direction yields for activities {A, B, C, and F} the allocation percentages of \( \{2/20 = 10\%, 4/20 = 20\%, 6/20 = 30\%, 7/20 = 40\%\} \). For the available CF = 6 workdays
these percentages yield $DF = \{0.6, 1.2, 1.8, 2.4\}$ workdays, which are rounded using accountant rounding to $DF = \{1, 1, 2, 2\}$ workdays of distributed float, respectively. Note that fortunately all critical subcontractors receive at least one workday of $DF$ and that no scaling is necessary, as the $DF$ sum correctly to $CF = 6$ workdays. But had the example chosen e.g. $CF = 4$ workdays instead (or used another rounding mechanism), these percentages would have yielded $DF = \{0.4, 0.8, 1.2, 1.6\} \approx \{0, 1, 1, 1\}$ and would have required scaling. Non-critical activities obviously do not receive any ownership in the $DF$, but could seek to purchase some in case they are delayed and become critical. Time Value of Money can be ignored for short durations, e.g. under 30 days, so that $r^* - t$ deconstructs to simply $T$ (here set to 1 to render $T$ moot) in Equations 1 through 3 and $r$ is unnecessary. The risk neutral probabilities $d_1$ and $d_2$ then translate to a single $\sigma$, which is the risk specific to a project or schedule, leading to a shortened valuation per Equation 4. The current spot price $x$ of the asset, herein $DF$, is quantified as the cost to increase ongoing operations to meet the as-planned completion. This directly corresponds to costs that are experienced beyond the as-planned duration. Obviously, the only differences to continue work are the extended overhead (OH) and general conditions. Added material and labor costs to complete the activity are equal, whether it extends beyond the as-planned duration or uses acceleration. Real options theory models uncertainty or volatility as $\sigma$. Yet no ubiquitous approach exists in construction; values for $\sigma$ still are based on rules-of-thumb. Uncertainty can be extrapolated by assimilating schedule uncertainty, cost volatility, and price fluctuations, and comparing them to other industries and modeled cash flows. A suitable analogy is the oil and gas industry, which strongly depends on construction services and has available historic values. The volatility of gas and oil prices over 25 years range from 30% to nearly 65% (Piesse and van de Putte 2004). Assuming $\sigma$ between 25% and 50% thus appears reasonable. Within construction schedules it can be defined as the activity specific potential to need float. It addresses the ability of each activity to meet the as-planned duration and considers its respective position in the sequence (i.e. later activities will face greater uncertainty for beginning and completing their respective work as planned). Values for $\sigma$ of the example as listed in Exhibit 1 are used to calculate the price at which float should be traded by critical activities resulting by applying Equation 4.

\[
w(x, t) = x \cdot e^{\sigma \sqrt{T}}
\]

(4)

Exhibit 1: Extended Activity List as Input for Schedule Example

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration</th>
<th>Successors</th>
<th>Cost</th>
<th>Daily Gen. Cond. + OH Cost</th>
<th>Variability $\sigma$</th>
<th>Daily Float Trading Price $w(x,T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>B, D</td>
<td>$5,000$</td>
<td>$750$</td>
<td>0.10</td>
<td>$830$</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>C, E</td>
<td>$18,000$</td>
<td>$500$</td>
<td>0.15</td>
<td>$580$</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>F</td>
<td>$32,000$</td>
<td>$650$</td>
<td>0.40</td>
<td>$970$</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>E</td>
<td>$20,000$</td>
<td>$600$</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>F</td>
<td>$9,000$</td>
<td>$700$</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>F</td>
<td>8</td>
<td>-</td>
<td>$16,000$</td>
<td>$800$</td>
<td>0.50</td>
<td>$1,320$</td>
</tr>
</tbody>
</table>

Exhibit 2: Schedule Example with Distributed Float

(Adapted with permission from Halpin and Woodhead 1998, p. 113)

**Analysis**

Daily Float Trading Price $w(x,T)$ in Exhibit 1 represents the price at which an activity should seek to exchange float to mitigate its delay rather than accelerating. The call price for purchasing float should be at or below the calculated value of the activity that needs float; the put price at which an activity should consider selling float should be above...
its option value. For example, assume that 9 days have elapsed, activities A and B were completed as-planned, and Activity C is halfway through its 6 days but remains 8 days from completion. Activity C is in the position of the last of the three aforementioned float trading scenarios. Activity C needs 5 days beyond its as-planned duration. As no float has been exchanged thus far, and all option values for activities already completed are below the option price for C, all DF should be acquired from A and B. Combined with that owned by C for a total of 4 days, one additional day of float is needed. This can be acquired from F at a price premium ($1,320 per day versus a call price of $970), or implementing acceleration will be required to overcome the remaining one day of delay expected by Activity C.

Conclusions and Recommendations

Float awaits being used in the most beneficial way. This paper contributes to the body of knowledge by outlining a feasible path toward converting float into a tradable commodity. In particular, project (or contract) float is identified as the sole opportunity for critical activities to acquire extra flexibility and mitigate risk. Such critical subcontractors are thus placed into a better position of sharing in a manner that benefits them and the project. Real options present a particularly promising way to obtain such monetary value. Future research will examine larger networks schedules with branches and merges in their critical path, review the issue of upstream versus downstream position on it, and explore under what circumstances non-critical activities might gain an advantage from DF actually being consumed.

References


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