Modeling Measures of Float Monetization for Quantitative Risk Management of Construction Projects

Richard C. THOMPSON, Jr.¹ and Gunnar LUCKO²

¹ Department of Civil Engineering, Catholic University of America, 620 Michigan Avenue NE, Washington, DC 20064; email: thompsrc@cua.edu
² Department of Civil Engineering, Catholic University of America, 620 Michigan Avenue NE, Washington, DC 20064; PH (202) 319-4381; FAX: (202) 319-6677; email: lucko@cua.edu

ABSTRACT

Risk is inherent in construction projects of all types and sizes. It has many causes and its effects manifest themselves in various forms. Most recognizable are impacts on the schedule, where they reduce float, i.e. its flexibility to absorb delays and thus mitigate risk. Regardless of the nature of risk, it is measurable by its effect. Several important issues arise, including how to allocate float fairly and equitably among subcontractors, valuing it for trading it before it expires, and using so that optimally mitigates the risk of the overall project. This research therefore explores one of these three parts of an improved risk management methodology, valuation. It uses real options, which model opportunities – not obligations – for decisions on tangible assets, to price allocated float for trade among critical project participants. The binomial decision tree or lattice of real options is analogous to junctures within network schedules. At such points participants may use or trade non-expired float to reduce delays along or near a critical path, or reserve it to hedge against future risk until it expires. In other words, it functions like put and call options. Overall, this study extends theory in construction risk management by pricing tradable float.

INTRODUCTION

Wherever risk and uncertainty reside, flexibility has value (Trigeorgis 1996). Placing a monetary value on flexibility depends on the source of the underlying risk, the point when appears, and the venue to which it belongs. Risk in network schedules, widely used the construction industry, is most quantifiable most by its impact on the schedule, as measured by consuming float – an ability to absorb delays without impact on its deadline. Valuation is the process of estimating the current worth of an asset, e.g. investments in marketable securities like stocks, options, or business enterprises, intangibles like patents and trademarks, or liabilities like bonds (WebFinance 2011). Analogously, the major question in this study is how to establish the value of such a flexibility in schedules. This research, the third in a three-part approach on how risk can be quantified, priced, diversified and mitigated, therefore focuses on pricing said flexibility in schedules. It extends the notion of the seminal article Total Float Traded as Commodity (de la Garza et al. 1991) by applying real options, a tool of corporate finance and capital budgeting. The following sections describe the background of real options, establish the need for float valuation, describe how valuation is performed with real options, and illustrate the new approach with a worked schedule example.
LITERATURE REVIEW

In 1983 the future chair of the U.S. Federal Reserve posited that uncertainty can increase the value of delaying decisions (Bernanke 1983). Investment decisions were examined under two assumptions: (1) Irreversibility, i.e. some decisions cannot be undone or substantially changed without incurring great or even prohibitive costs; (2) Information, i.e. not all information that is relevant to a decision may be available immediately, and new and/or better information may become available in the future. It was concluded that postponing a decision, while maintaining the ability to commit at a later time, is desirable because it allows deciding only after important information is revealed. This concept is the foundation of options theory in finance and budgeting.

Several formal techniques are used in capital budgeting decisions. They rely on measuring cash flows into and out of a company or project and include two types, discounted cash flow methods and non-discounted cash flow methods (Ross et al. 2009) as listed in Table 1 (Laudon and Laudon 2011). Their difference lies in whether or not they fully consider the Time Value of Money, i.e. the principle that amounts of money will change value between two points in time because of the interest rate \( r \). When faced with choosing from alternative options, managers may use one or multiple techniques to determine which one to undertake. Each technique focuses on a specific aspect of the decision and has advantages and disadvantages. In a world of complete certainty, net present value is most suitable. However, uncertainty in future cash flows and the interest rate \( r \) require subjective forecasts. Proper evaluation thus has to consider measures of uncertainty in advanced techniques (Quiszpe-Asin 2008).

Table 1: Comparison of Existing Techniques for Capital Budgeting

<table>
<thead>
<tr>
<th>Technique</th>
<th>Time Value of Money</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Present Value</td>
<td>Yes</td>
<td>( \Sigma ) of all flows discounted at ( r )</td>
</tr>
<tr>
<td>Cost-Benefit-Ratio</td>
<td>Yes</td>
<td>Total benefits / total costs</td>
</tr>
<tr>
<td>Profitability Index</td>
<td>Yes</td>
<td>( PV ) of inflows / initial investment</td>
</tr>
<tr>
<td>Internal Rate of Return</td>
<td>Yes</td>
<td>Iterated ( r ) to achieve ( NPV = 0 )</td>
</tr>
<tr>
<td>Modified Internal Rate of Return</td>
<td>Yes</td>
<td>( \sqrt{FV \text{ inflows} / - PV \text{ outflows}}, ) distinguishes invest, finance: ( r_i, r_f )</td>
</tr>
<tr>
<td>Payback Period</td>
<td>No</td>
<td>Investment / average inflows</td>
</tr>
<tr>
<td>Return on Investment</td>
<td>No</td>
<td>((\text{Gain} - \text{cost} / \text{cost}) ) of investment</td>
</tr>
<tr>
<td>Average Rate of Return</td>
<td>No</td>
<td>Average profit / average investment</td>
</tr>
</tbody>
</table>

Options

In its broadest sense, an option is defined as “a security giving the right to buy or sell an asset, subject to certain conditions, within a specific period of time” (Black and Scholes 1973, p. 637). In the financial arena, it creates a contract to buy or sell a security, i.e. a stock, bond, currency, or commodity, at an agreed-upon price during a specified period or on a specific future date. Stock options were first traded in 1973 (Hull 1999) and nowadays are common in financial markets and portfolio theory.

The price paid for an option differs from its value, which is the ultimate profit to its owner should the transaction be completed. Its price derives from the difference between the exercise price and current asset value plus a premium for the remaining
time the option expires. The value to its owner, on the other hand, is theoretically unlimited. The financial risk of purchasing an option is limited to the losing its price, as an owner of an option is not obligated to make financially detrimental transaction.

**Definition of Real Options**

Real options create value under uncertain conditions. Their theory contributes to managerial decisions in three parts (Amram and Kulatilaka 1999): (1) Options are a contingency that offer an opportunity to decide after events unfold; (2) Valuations are aligned with market valuations –comparing alternatives and transaction costs on-par; and (3) Option analysis supports proactively managing strategic investments after identification and valuation, so that investments can still be reconfigured beneficially.

The hedging right provided by real options is an actionable ability to respond to change (Ku 1995), with a positive correlation of flexibility and value (Hirshleifer and Riley 1992). Real options are categorized whether they act ‘on’ or ‘in’ projects. The former type affords a maneuver (Chambers 2007), e.g. (1) Acquire (buy or start) project; (2) Divest (sell or abandon) it; (3) Expand its size; (4) Contract its size. Real options ‘in’ a project, conversely, are much far more diverse, complex, and difficult to identify and appraise. Applying them must consider the complexity of the project or engineered system. Their flexibility is provided through the design of the system itself (de Neufville et al. 2004) and are of particular interest to uncertain, lengthy, and complex projects or systems with economies of scale (Wang 2008, Roos 2004). These characteristics support the approach to apply option pricing to construction projects.

**Difference to Financial Options**

A financial option is defined as the right to buy or sell an underlying asset at a specific price during or after a period of time. A real option is a special case thereof that requires a capital investment in a project or system. Moreover, whereas the value of the former is based on information that is readily available to all interested parties, real options generally rely on information that is proprietary or available to decision-makers only (Copeland and Tufano 2004). Furthermore, the value of the underlying asset of a financial option is typically known from comparable ones, but a comparable asset for a real option may not exist or its value is difficult to establish. Finally, their terms on e.g. right and period are often less clear than their very prescriptive financial counterpart that is readily executable in accessible financial markets. Besides these uncertainties, real options are compound, wherein sequential decisions may uncover new options, rather than the asset itself, which further hampers clarity. Financial and real options thus differ in terms of identification, valuation, and complexity. Their purpose remains the same, purchasing a right to delay costly or irreversible decisions.

**Option Pricing Techniques**

Options appear in form of puts or calls, i.e. the future right to sell or buy an asset, respectively. They can be evaluated for pricing by several mathematical models that have been published in the financial and economics literature. Table 2 identifies various well-established option valuation methods that are applicable to financial and real options and briefly describes the individual techniques that establish their prices.
### Table 2: Option Pricing Methods

<table>
<thead>
<tr>
<th>Name</th>
<th>Methodology</th>
<th>Application Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black-Scholes Model</td>
<td>Partial differential equation</td>
<td>$C(S, t) = N(d_1)S - N(d_2) \cdot Ke^{-rT}$ where:</td>
</tr>
<tr>
<td>(Black and Scholes 1973)</td>
<td></td>
<td>$C(S, t) =$ Call value stock $S$ over time $t$;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N(d_1)$ = Call option cost;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N(d_2)$ = Risk neutral probability of $S &gt; K$;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Ke^{-rT}$ = Present value of option cost.</td>
</tr>
<tr>
<td>Finite Difference</td>
<td>Numeric method: May be partial differential equation based</td>
<td>A function of time and underlying asset price. Calculated recursively. Daily option value determined by interpolation.</td>
</tr>
<tr>
<td>Monte Carlo Simulation</td>
<td>Iterative</td>
<td>Generation of all random possible price paths and associated payoffs. PV of average payoff across simulation runs determines option value (Crack 2004).</td>
</tr>
<tr>
<td>Binomial Valuation</td>
<td>Decision analysis: Binomial tree or lattice</td>
<td>Determination of upward and downward decision movements based on volatility and probabilities for movement per Equations 1 and 2 below (Cox et al. 1979).</td>
</tr>
</tbody>
</table>

### NETWORK SCHEDULES AND FLOAT

This research focuses on network schedules that have been analyzed with the traditional critical path method (Kelley and Walker 1989) that adds activity durations along paths of dependency between their starts and finishes. Float appears in various forms therein, including the well-known total float as the difference between earliest and latest dates of activities just before impacting the overall project duration, and free float as the difference of latest finish of a predecessor activity and the minimum earliest start of any successor. This research focuses on another type of float that is barely recognized and reveals a central paradox of traditional network scheduling: By definition, non-critical subcontractors have float, but the critical ones have none, even though they need it most. This contract float has been newly defined as the difference between the calculated project finish and the contractual deadline. When it has been apportioned to individual critical subcontractors according to the method proposed by Thompson and Lucko (2011) it is called distributed float. Note that allocation gives ownership of float days to a specific subcontractor, but does not advocate to delay a start or extend a duration. Rather, it is a contingency and, if unused, can be sold.

### Relationship of Risk and Float

Construction projects evolve over time, subject to internal and external risks that cannot be completely forecasted. A general contractor and various subcontractors participate in a complex decision-making process with constraints and uncertainties which determines if the project will finish on time. Planning and managing such an intricate process therefore must identify, assess, and select among multiple options (Ford et al. 2002). The ultimate quantification of uncertainty, or risk, is its realization in a change event. This, in turn, will consume float, thus reducing flexibility. Float therefore mitigates risk and increases opportunity (Thompson and Lucko 2011).
Need for Float Valuation

Construction managers must mitigate uncertainties before they proceed, but knowledge of future conditions is either unavailable or inadequate. Uncertainty has negative connotations like schedule delays and budget losses. Project managers tend to limit their efforts to the mitigation of uncertainty and its undesirable impacts while ignoring hidden and unexploited opportunities (Bhargav 2004). They are aware of potential benefits in some cases, e.g. in ‘fixed-price’ versus ‘cost-plus’ contracts, but lack strategies and tools to systematically evaluate such behavior of projects: “The ubiquity and potency of dynamic uncertainties [i.e. costly suboptimal options] require that they be managed effectively if all the value in a given project is to be developed and captured” (Ford et al. 2002, p. 344). This can be achieved by “[a] real options approach in construction projects [that] improves strategic thinking by helping planners and organizers recognize, design, and use flexible alternatives to manage uncertainties” (ibid., p. 346). This notion was reinforced by Boute et al. (2004, p. 9) as “[i]t is inherently clear that the longer the contractor waits, the more additional information he obtains and thus the more valuable the option will be.” In fact, each construction project, as well as individual activities therein, can be considered as a real option. The question therefore arises how to apply them to network schedules.

Applying Real Option Methodology

Real options analysis structures the problem so that its various uncertainties and contingent decisions are represented by a decision tree or lattice (Bhargav 2004). Network schedules present distinctive structural parallels. Whereas a decision tree is a sequence of decision nodes with probabilities for independent and mutually exclusive branches that emanate from decision nodes are analogous to decisions, a schedule has dependencies of predecessor and successor activities, which require active decision-making at their starts and finishes. The particular analogy arises through ‘deciding’ to expend float in a schedule, which occurs due to involuntary delays or as a deliberate way optimize its performance. In other words, activities incur a binomial decision: (1) Expend float if available to mitigate schedule impact; or (2) Do not expend it, but accept the delay penalty or accelerate the activity by another means at additional cost.

Real options analysis requires up to five types of input values. To newly apply binomial valuation to float valuation within network schedules, variables must have known or assumed values as follows (Frayer and Uludere 2001, Cox et al. 1979):

- Present value $S$ of underlying asset in monetary terms;
- Strike price $K$ or investment cost;
- Duration $\Delta t$ until the current opportunity disappears;
- Risk-free rate of interest $r$ for Time Value of Money;
- Uncertainty or volatility $\sigma$ as a probability in percent.

A decision tree is analyzed over multiple periods for an increasing number of options along its paths. Analogously analyzing a network schedule treats all activity finishes as binomial decisions, i.e. ‘on time’ or ‘late’. Each delay equals one period. Figure 1 shows such decision tree, where $S$ is a dollar value of the asset that increases or decreases with the factors $u$ or $d$ for upward or downward move, respectively, from
period to period. To simplify it is assumed that the order of moves is interchangeable, i.e. decisions do not depend on prior decisions in a changing environment. In that case, a decision tree becomes the decision lattice as shown in the middle of Figure 1. Another assumption may be employed, that decision periods are often shorter than typical interest-generating financing periods, so that the Time Value of Money might be neglected. This assumption is applied to the following exemplar, where expending is measured in workdays, but financing accrues interest each month or even longer.

![Decision Tree, Lattice, and Example](image)

**Figure 1: Decision Tree, Lattice, and Example (Brandão et al. 2005)**


Equations 1 and 2 provide the model by Cox et al. (1979) to calculate $u$ and the probability $q$ of said upward move that depends on several inputs, including $u$. For an example of an asset of $S = \$100$ value whose opportunity disappears in $\Delta t = 0.5$ years with $r = 2.5\%$ time value of money over that period and a volatility of $\sigma = 0.2$, Equation 1 provides the factor $u = 1.1519$ and its inverse $d = 1/u = 0.8681$. Equation 2 yields the probability of the upward move $q = 0.51$ and $1 - q = 0.49$ for the downward move. Considering the time value of money with $r = 2.5\%$ over one period reduces $S \cdot u = \$100 \cdot 1.1519 \cdot (1 + 2.5\%)^{-1} = \$112.38$ and $S \cdot d = \$100 \cdot 0.8681 \cdot (1 + 2.5\%)^{-1}$.

$$u = e^{\sigma \sqrt{\Delta t}}$$  \hspace{1cm} \text{(1)}

$$q = \frac{1 + r \cdot \Delta t - d}{u - d}$$  \hspace{1cm} \text{(2)}

**EXEMPLAR**

Figure 2 shows the network schedule for the exemplar with contract float of 18 days until the contractual duration of 90 days, which becomes distributed float ($CF$ and $DF$) per the dashed boxes along the critical path (Thompson and Lucko 2011). It is marked with thick lines and boldface font. Early and late starts and finishes are $ES$, $EF$, $LS$, $LF$, total and free float are $TF$ and $FF$. Costs of the seven critical activities *Mobilization*, *B*, *C*, *F*, *I*, *L*, and *Turnover* respectively are $\$145$k, $\$25$k, $\$195$k, $\$110$k, $\$310$k, and $\$25$k to expand the previously published values. Daily general conditions and overhead costs for them are $\$1.0$k, $\$4.5$k, $\$625$, $\$2.0$k, $\$1.5$k, $\$3.5$k, and $\$1.0$k.
Analysis

Calculating the value of float for exchange between two activities requires selecting one of the aforementioned valuation methods. Bhargav (2004) confirms that the real option component of decision analysis that takes into account the value of flexibility and uncertainty of is best served by adapting the decision tree structure. Accordingly, the real option pricing of tradable float in this paper will follow the binomial decision structure of Cox et al. (1979). The real option value (that herein is considered a call option, the right to acquire an asset in the future) within a decision tree is calculated at each node as expressed by Equation 3, which reduces to Equation 4 when applied to the valuation of float and by the substitution for $u$ from Equation 1:

$$C(X,t) = \frac{(S-K)u}{(1-r_f)^t}$$  \hspace{1cm} (3)

$$C(X) = \frac{S\cdot e^\sigma}{(1-r_f)^T}$$ \hspace{1cm} (4)

For projects of short duration with less than one year ($T < 1$), the chosen time unit, including a discount factor may not be appropriate or too cumbersome. Such simplification is accomplished mathematically by setting $T = 0$, so that $(1 - r_f)^T$ defaults to 1, thereby eliminating any forward discounting. Except for the Time Value of Money ($r_f$) and timeframe ($T$) for option valuation, which are easily discernable, specific consideration and definition is required for the other three real option inputs:

- **Investment Cost ($S$):** De la Garza et al. (1991) define the costs associated with trading float as the difference between late costs (LFC) and early finish costs.
(EFC). However, it is difficult to calculate this difference for all activities as some may never experience LFCs. A more consistent approach would be to define it as the additional cost of modifying operations to meet the as-planned duration: Namely, the daily extended overhead and general conditions costs;

- **Uncertainty / Volatility (σ):** Project uncertainty and/or schedule volatility is not a metric common to the construction industry, despite a notoriety for time and cost overruns (Creedy *et al.* 2010, Shane *et al.* 2009, Georgy *et al.* 2000); with schedule volatility (projects experiencing delays) the standard deviation across multiple types of projects amounting to 38.7% (Flyvbjerg *et al.* 2002). Drawing from related industries like oil and gas production (Piesse and van de Putte 2004) and development, and construction material price (Lindsey *et al.* 2011), volatility ranges from 2% to 40%. Therefore, assuming the range of project uncertainty σ in the exemplar from 25% to 50% appears reasonable;

- **Present Value of Investment (K):** Not considered herein because $T < 1.0$. Float has no premium associated with its acquisition beyond its real option value.

Assuming a project uncertainty of $σ = 0.40$ results in $u = 1.4914$ from Equation 1 and yields call option values $C(X, T)$ for critical activities Mobilization, B, C, F, I, L, and Turnover respectively as $1,492, 6,713, 932, 2,984, 2,238, 5,221$, and $1,492$; e.g. $C(F, T) = S(F) \cdot u = 2,000 \cdot 1.4914 = 2,984$. This represents the price at which the distributed float should be sold by activity $F$ or at which it should acquired prior to consideration of engaging in any schedule acceleration methods.

**CONCLUSIONS**

This research began by linking real options theory from capital budgeting and finance and the Bernanke (1983) precept that waiting for additional information has inherent value in decision-making with network schedules and their inherent property to mitigate risk, float. The literature has confirmed that the binomial decision process and real option valuation method is applicable to the pricing of distributed float that has been allocated across critical schedule participants by previous research. By way of an exemplar it has been demonstrated that call option values specific to activities can be derived from the general conditions and overhead costs (which here is the only unique option investment cost) with the use of $σ$, the project uncertainty or schedule volatility. For practical application, its actual value would need to be assessed by the individual critical subcontractors to aid them in decision-making of whether to sell or buy float, depending on their performance, or having to resort to costly acceleration.

**FUTURE RESEARCH**

This research is one of three elements of a larger research approach on risk management, specifically its location, quantification, and pricing, to mitigate negative impacts in network schedules. Through this pricing effort, a shortcoming at the core of the available metrics of the construction industry was discovered: Namely, the lack of common methods, measures, and historic data on project uncertainty, especially at the activity-level. Additional work in this area is urgently needed. Moreover, other conceptual analogies on risk management that are in practical use in other industries or exist in the academic realm need to be explored to potential beneficial use in the
construction industry. Furthermore, a predictive modeling mechanism to analyze risk in network schedules should eventually lead toward a market whose standard contract clauses allow project participants to competitively trade their critical resource, float.

REFERENCES


