Modeling Measures of Float Allocation for Quantitative Risk Management of Construction Projects

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Abstract: Risk is ever-present and manifests itself in different ways in construction projects of all sizes and complexities. Its most recognizable and quantifiable form is schedule risk, which is related to float within a project, i.e. its flexibility to absorb delays. Allocating, owning, valuing, and expending such float in network schedules has been debated since the inception of the critical path method. This research investigates the first element of a three-part approach that models how float may be mitigated by allocating, correlating, and valuing it so that it may eventually become traded as a commodity, an unrequited concept in construction engineering and management. Mathematical voting models that are based on the power of a single entity to change the outcome of an election are explored as analogies for the interactions of participants in construction projects: Disparate entities, here subcontractors, participate in a complex decision-making process subject to constraints and uncertainties as to whether the project will be on time. Risk measurement is explored through the Penrose-Banzhaf ‘square root law’, which allocates voting power based on the square root of the population where voting rights are not equally divided. It newly adapts this theory to allocate float among participants and their activities, who likewise have unequal individual durations, values, and positions in the network, based on the square root of their respective population elements: Schedule duration or contract value. Recognizing the impact of each entity on the project outcome represents the first element of the model – allocating tradable float.

1. INTRODUCTION

Risk is ever-present in construction projects. Although much has been written about its cause, effect, mitigation, avoidance, and transfer through numerous techniques, programs, contractual terms and conditions, or insurance vehicles, there exists no widely accepted standard practice for its allocation or valuation. Risk, beyond its physical appearance through e.g. weather conditions, project-related mishaps, or other external influences (project changes, building code changes, non-conforming work, etc.), is most recognizable and quantifiable by its impact on a project’s schedule, as measured by consumption of float.

This research, the first in a three-part approach to modeling how systematic risk can be quantified, priced, diversified and/or mitigated, and the development of a prediction method for where risk is likely to reside, is focused on the allocation of risk among a construction project’s participants. It is the first step toward realizing the concept of Total Float Traded as Commodity (de la Garza et al. 1991) and has its roots in social decision-making models. The following sections describe the background of how what types of risk exist and how it impacts network schedules, decision-making models and especially voting models, then derive the central new analogy of subcontractors acting similar to voters in a decision-making process of whether or not to expend float, review existing approaches to float ownership, provide an exemplar that illustrates how the new approach can be used to allocate float, and draw conclusions on its contributions.

2. BACKGROUND

Risk in general is the “effect of uncertainty on objectives”, regardless of whether it is positive or negative (ISO 2009). However, in practice the negative aspect dominates and risk is specifically understood as the
“[p]robability or threat of a damage, injury, liability, loss, or other negative occurrence, caused by external or internal vulnerabilities, and which may be neutralized through pre-mediated action” (Webfinance 2011).

2.1 Risk Categorization

Risk is represented by, can be found in, or arises in four basic categories (Klemetti 2006, p. 14), including “pure risks (e.g. hazards and weather conditions [including environmental disasters]), financial risks (e.g. cash flow or credit risk), business risks (almost anything that can happen in a project) and political risks, which refer to the certain political environment and risks [e.g. changes in governments, laws, or regulations] that are caused mostly by extreme conditions, such as… war” or cross-cultural hostilities.

Further studies (e.g. Baloi and Price 2003) classify different types of risk for construction projects into the categories of technical, construction, legal, natural, logistics, social, economic, financial, commercial, and political risk. Risk, and in-particular risk that is related to construction activities and its manifestation within network schedules, can be mitigated. Four primary strategies exist to accomplish this: Transfer – shifting the risk to another party; avoidance – forestalling the risk; mitigation – reducing the negative effect of the risk; and/or acceptance – realizing some (or even all) of the negative consequences of an identified risk.

2.2 Translation to Network Schedules

The unifying factor among all risk classifications is the potential to impact a construction project schedule; herein referred to as schedule risk: The need for a project (or more precisely its schedule) to exhibit flexibility to absorb externally-influenced and/or externally-originating delays. This is inextricably linked to and defined as float, which can be impacted by risk in two ways: Positively, when the duration required to complete individual tasks is less than the schedule identifies, causing it to advance or finish quicker, or negatively, when task durations are exceeded and delays of the project completion should be expected.

Unfolding from risk identification and classification is the source or cause of construction project risks and uncertainties that can have negative impacts. Various primary factors for the cause of significant delays on construction projects were listed, including e.g. owner interference, inadequate contractor experience, financing and slow payments, labor productivity, slow decision-making, improper planning, materials quality or shortages, and subcontractors (Odeh and Battaineh 2002). These factors represent risks that can be viewed through network scheduling. Decision-making models can express how risk is handled.

2.3 Decision-Making Models

The way in which individuals, groups, organizations, and political entities combine their preferences, needs, and choices into a decision or overcome a perceived risk is a question about which there has been considerable speculation, research, and mathematical modeling. In democratic capitalist societies there are essentially two primary methods by which such choices or decisions are made (Arrow 1964): Voting – typically used to make political decisions; and the mechanisms of the marketplace – typically used to make economic decisions. Groups may engage in majority votes with or without veto power, be ruled by dictatorial leadership models including unilateral decisions by fiat, form collective bargaining agreements, evaluate options through mathematical models, decide by governing board and consensus, or may employ economic value-based methods. Such group decisions, or social choices, are made in many ways with mechanisms unique to the specifics of the group: Larger and more complex groups tend towards voting and elections; smaller ones favor committee structures and consensus (Lieberman 1971).

Irrespective of specific decision-making approach, the issue of combining individual preferences into socially acceptable outcomes or choices has both normative and descriptive aspects. The normative aspect of the social choice question is described in its basic form as how should groups ideally combine individual preferences into a sensible and consistent decision to produce results. The descriptive question is how do individuals meld their preferences into an agreeable group decision. The commonality between these diverse aspects of social choice is the conflict resolution resulting from contradictory preferences, traditions or customs. The voting method and those decisions that are left to the marketplace represent
the amalgamation of individual tastes and preferences through the decision-making process, yet both of them involve a collective choice among a limited range of alternatives or opportunities for the decision.

The issues of allocating, owning, valuing, and expending float in network schedules have been debated since the inception of the critical path method and are the contradictory preference at hand. The choice to expend float is the ultimate conclusion or 'decision' reached in the risk management progression in any network schedule-based construction projects. The various determining processes which the members of the group or organization – the collection of subcontractors and the general contractor as applies to this research – undertake has six general elements (Lieberman 1971) thereafter: Distribution of power, joint welfare, bargaining and coalition processes, individual differences and characteristics, group phenomena, and the previous experiences and commitments of the group (including possible future interactions).

The preponderance of these elements can be found in the decision to expend float, wherein the foremost concern is the distribution of power or the lack thereof. One of the most common processes by which such social choice is made, the manner in which the collective integrates individual preference into a decision, is by voting (Birnberg and Pondy 1971). Within the distribution of power through voting three characteristics emerge as the determining elements upon which rests the satisfaction of the participants: Size – the number of persons and of issues; distribution – how preferences on an issue are distributed over individuals; and correlation between the preferences of one individual and others about an issue.

2.4 Voting Practices

Decision rules and the exercise of control through voting power is very complex. Some frameworks for voting power consider determining elements of size, distribution, and correlation, with the bicameral form of the U.S. Congress, a representative democracy, and the European Union, a federation of independent countries, being most contemporary and comprehensive federal systems. Neither the representation nor decision-making embodied by the U.S. democratic republic or the EU federation are directly analogous to network scheduling, given the indirect ('representative') nature of their decision-making rules. There remains a gap between the constituency and the vote for a decision. This prevents a direct correlation to the inner workings and decision-making modes that are found in network schedules.

Where voting practices become apropos to this research is that mathematicians have examined voting process at the supranational level (Scientists for a democratic Europe 2004) to validate their tenets and search for appropriate alternatives to more effectively allocate voting representation by the use of predictive and distributive mathematical models. The following sections summarize their basic concepts.

2.5 Analytical Voting Models

Voting models may provide insight into allocating float, the a priori means of mitigating risk within network-based schedules. The seminal work by Penrose (1946, p. 53) provided an insightful approach to voting power (which was later independently recreated by Banzhaf (1965) in form of a so-called ‘power index’):

“In general, the power of the individual vote[r] can be measured by the amount by which his chance of being on the winning side exceeds one half. The power, thus defined, is the same as half the likelihood of a situation in which an individual vote can be decisive – that is to say, a situation in which the remaining votes are equally divided upon the issue at stake. The general formula for the probability of equal division of n random votes, where n is an even number, approaches \( \sqrt{2/n\pi} \) when n is large. It follows that the power of the individual vote[r] is inversely proportional to the square root of the number of people in the committee.”

The essential principle is that the number of voters does not matter directly. Rather, such representative systems use a multi-level election (vote) to make a decision, where at each level the majority rule creates a 'winner takes all' outcome that is directly comparable to the first-come, first-serve consumption of float.
2.6 Contribution of Lionel Penrose: Square Root Law

The work of British psychiatrist and mathematician Lionel S. Penrose (1898-1972) has been recognized as a fundamental work on the measurement of voting power. His short paper proposed a probabilistic measurement of hypothetical votes in the newly formed United Nations General Assembly. It argued that the equitable distribution of voting power should be proportional to the square root of its population.

To overcome the problem that “if large and small nations have equal voting powers, the spokesmen for small nations are felt to be too significant, and artificial rules about the meanings of votes and vetoes have to be constructed to redress the balance” (Penrose 1946, p. 56) and to achieve an optimal two-tier voting system (where a set of constituencies of various size elect one delegate each to a decision-making body) in which every citizen of every country has the same potential voting power, he first considered a direct election in a state consisting of n voters and proved that the voting power of a single citizen decays as \(1/\sqrt{n}\), assuming that the votes are uncorrelated. To compensate he suggested that the a priori voting power of each representative in the voting body should be proportionally to \(\sqrt{n}\), which makes the voting power of citizens in all states equal. Ultimately, the two factors cancel each other out as \(1/\sqrt{n} \cdot \sqrt{n} = 1\).

The reason for assigning voting weight in proportion to the square root \(\sqrt{n} = n^{0.5}\) of the population n of each country is that this value strikes an ideal balance exactly between the aforementioned ‘one vote per country’ (proportional to \(n^0\)) versus ‘one vote per person’ (proportional to \(n^1\)) (Pöppe 2007). This became eponymously known as the Penrose-Banzhaf ‘square root law’, which has inspired this new research.

3. ANALOGY TO CONSTRUCTION NETWORK SCHEDULES

3.1 Correlation of Voting Measures to Float

Disparate entities, herein called subcontractors, participate in a complex decision-making process that is subject to constraints and uncertainties of whether the project will be on time and within budget. As votes comprise an election, so do risks contribute to a project. Building upon social choice and decision-making, voting typifies a possible determinant for the behavior of those analogously participating in construction projects and provides a potential mitigation approach to lessen the impacts of risks on network schedules.

Despite what initially may appear to be an insurmountable theoretical difference between political voting systems and project management practices, numerous conceptual analogies can indeed be identified. A federal system whose elected representatives campaign and vote toward a decision (for or against a bill), is analogous to a network schedule (on time and within budget or not), wherein construction work is planned and performed by different subcontractors. Both elections and projects are modeled as binary decision-making processes. Whereas voters have a quantifiable influence – their voting power – on the decision, so do subcontractors influence whether the project will be on schedule and within budget or not.

Developing the voting analogy at the structural or formative level, conceptual comparisons or correlations of the participants, actions and outcomes of an election can be mapped to the participants within a project and its network schedule. For example, in this research an election is mapped to the overall construction project, the aforementioned federal system is analogous to the network schedule, and other elements of the voting or electoral process are mapped as follows (not an exhaustive list): Representation ↔ non-self-performing or outsourcing; campaigning ↔ planning or estimating; ballot ↔ schedule and budget; voting weight ↔ activity value or duration; voters ↔ subcontractors; vote (yes/no, for/against) ↔ float-related decision (yes/no, expend/accelerate); win or lose ↔ on-time / within budget or delayed / over budget.

Furthering the analogy to voting, Gelman et al. (2002, p. 429, emphasis added) posit that in evaluating such voting models, patterns, and power “[t]he next step is “give a dependence structure to the voters’ probabilities…” [where it] makes sense to build this dependence upon existing relationships among the voters”; a reference to the geographic structure of the U.S. with states, Congressional districts, counties, cities, precincts, etc., or conversely, the ‘softer’ demographic of the multiplicity of constituencies forming...
the electorate. Herein such a dependence structure is the existing relationships between the collection of subcontractors and their risks and intricacies nested within a network schedule – in particular, float. Float is a measure of flexibility that reduces risk and increases opportunity. It quantifies the ability of an entire schedule or individual activities therein to absorb delays. Just as a vote is the vehicle by which decisions are made, so the decision to expend float is the analogous ‘vote’ within network schedules. The only significant conceptual difference between elections and projects is that activities in a construction project occur within a dependency structure per the network schedule, whereas blocs cast their votes all at the same time into the ballot box. This creates a challenge for investigating how these voting models can be converted into the basis for allocating float within network schedules in the construction industry.

3.2 Critical Path Method and Float Measurement

Project planning is strongly centered on the discrete scheduling of time. Network scheduling originated in 1956 when Kelley and Walker (1989, 1959) planned a factory with an early UNIVAC computer. Their critical path method (CPM) adds deterministic durations from the overall start forward along paths of fixed dependency and compares these early dates with subtracting durations backward from the overall finish for late dates. Links connect starts (S) and finishes (F) as either F-S, S-S, F-F, or S-F and may carry lags. It is not widely recognized that this recursive algorithm is a simplified case of linear programming (Dantzig 1955) used to solve a system of numerous equations with the general form of \( \text{start} + \text{duration} = \text{finish} \).

When a difference exists between the earliest and latest possible dates of an activity as computed by the forward and backward passes, float arises i.e., the activity has a flexibility to absorb delays. That is, there is an additionally available duration within which it can be completed before impacting successors or even the project finish. Zero float is considered (most) critical, hence the name of continuous path(s) through a network schedule that in turn gave the entire method its descriptive name that is well-known in practice.

Total Float is a byproduct of CPM calculations and represents the length of time an activity's finish date may be delayed without affecting the completion date of the entire project (de la Garza et al. 1991). Total Float is an attribute of the network path and is not assigned to any one specific activity and cannot be 'owned' by an individual activity. Its consumption will reduce Total Float times for any direct successor activities. Total Float is always shared with the other activities along the same path (Callahan et al. 1992).

Float is measured in multiple ways and defined differently depending upon the viewpoint, need, or relative position within a given network schedule. The most common and well-known definitions of float include:

- **Total Float**: Possible delay of activity \( i \) without impacting project end date, is therefore of interest for owner of the project. It is always shared by all activities along one path and thus also called path float;
- **Free Float**: Possible delay of activity \( i \) without impacting any of its successors \( j \), is therefore of interest for subcontractors of the project. It is not shared, but is unique to the individual activity. Also called real, internal, or activity float. Interestingly, the sum of Free Float along one path is the Total Float. As such, it is ‘stricter’ than Total Float in that it is always zero if the Total Float already is zero (critical).

3.3 Float Ownership and Allocation Approaches

Issues and constraints that influence float allocation have been identified in previous research (e.g. Al-Gahtani 2009). Numerous approaches to the ownership, allocation, control, and expenditure of float can be found in practice to compensate for the project risks and their schedule and cost impacts. They can be classified into three main categories: Owner entitlement, contractor entitlement, or project entitlement.

Irrespective of the individual approach to float, it is the terms of the contract that allocate the risk between participants (Prateapusanond 2003). Yet the actual risks per Section 2.1 should govern the allocation and exchange of float among the project participants. Ownership of such float should therefore be decided in advance to be allocated among the participant on the basis of their comparative risk levels (Al-Gahtani 2009). Such an approach decreases the likelihood that there will be disputes related to the use of float. However, as contracts are rooted in negotiations rather than in scientific-based approaches, rigorous mathematical models, or analysis, supplemental methodologies are necessary to address the schedule-based Total Float issues of ownership, allocation, and consumption, and the risks associated therewith.
3.4 Legal Perspective on the Ownership of Float

Total Float is recognized as a by-product of CPM calculations and may or may not exist if activities have not yet commenced, or if they were not started on the date initially set forth in the baseline schedule. It is also expiring if it is not consumed when it happens to be available and/or assigned to a particular activity. That is, it simply will disappear as time progresses and work ensues if it is not used whenever delays are incurred that require consumption of float to keep the project within the originally planned bounds.

Irrespective of the cause of the delay or the specific entity who is consuming float, case law supports the principle that float is available to all involved in the work, and is not the specific property of anyone – it “is an expiring resource available to all parties involved in a project” (Wickwire et al. 1991, p. 333, emphasis added). Public procurement contracts typically include language prohibiting designation of float for the specific and/or exclusive benefit of any one participant involved in the work. These clauses are also predicated upon the premise that the first entity utilizing the available float or a portion thereof to gain the benefits thereof (first-come, first-serve). Given this notion, entities with some available float would be free to use it for their specific benefit without any regard to the potential positive impact that it could have on other participants in the network schedule or to the overall project completion time in a ‘win-win’ fashion.

4. FLOAT / RISK ALLOCATION

Drawing from Penrose (1946), the ‘square root law’ represents a unique concept for the allocation of float (based on its aforementioned correlation with risk) in construction network schedules. To translate this to network schedules and determine whether there is a better methodology for allocating float among the participants, this research presents initial calculations, analysis, and conclusions by way of an exemplar.

4.1 Research Expectation

The overarching expectation is that this research depicts a float allocation method that addresses the ubiquitous treatment and understanding of the Total Float. It is a vanishing commodity that it is generally consumed on a first-come, first-serve basis and is not owned by any single entity (owner or contractor) or participant (subcontractor). The new approach seeks to define an equitable means for allocating float to the participants who are most in need of its flexibility: Critical network participants – those on the critical path who by definition have no float available. In addition, this research is expected to lay the foundation for, and first element of, the initially posited three-part approach to how risk can be quantified, priced, diversified and/or mitigated systematically and optimally in network schedules of construction projects.

4.2 Exemplar Development

Reviewing the body of literature and related previous research of the authors, a simple network schedule that has been used to depict network complexity and differing time calculation methods (Lucko 2005) is expanded upon to depict a project whose attributes and performance can easily, but accurately, portray the concepts under development, and lend credibility to its analysis, conclusion(s), and extension. Figure 1 graphically portrays its starts and finishes, durations, and dependency structure, or logic. To these input and output time-based parameters Table 1 adds a assumed set of activity costs. Their values represent what could be realistically expected in a small-sized project in any sector in the construction industry.

Beyond the cost data, further definition of project constraints is necessary for the exemplar to be sufficient for use herein and overcome an intrinsic shortcoming of network schedules that are analyzed with CPM: The CPM duration is limited to the activities contained therein, not to other extraneous conditions placed upon the end of the network. Specifically, contractual and other obligations of complexities that define the totality of the project work and duration do not extend beyond its 72-day-long duration. Therefore, besides addition of cost constraints to the exemplar, the overall contractual completion of the project is set at 90 days. This yields 18 days of float to be allocated between the calculated and the contractual finish dates.
4.3 Local versus General Total Float

Within this research, Total Float (TF) – the time by which an activity may be delayed or extended without affecting the total project duration, or violating a target finish date, requires further definition. Given the addition of 18 days to that 72-day-long duration, Total Float occurs in two distinct types – ‘general’ and ‘local’. In other words, the former is post-CMP duration float that relates to the overall completion of the project (hereinafter referred to as Contract Float) and the latter relates to specific adjacent activities within the CPM network. They need to be identified and addressed separately, because the former one has (potential monetary) value to the critical activities (those on the critical path), while the latter one does not.

4.4 Float Allocation Metrics

4.4.1 Contract Float Allocation

The addition of activity costs and Contract Float (CF) to the exemplar results in parallel paths for float allocation. The first is based on activity cost and its square root and the second is based on duration and duration square root. However, initial analysis of a preliminary Contract Float model concluded that its allocation is only applicable to critical participants. In this analysis this limits allocation of Contract Float to activities ‘Mob.’ (mobilization), ‘B’, ‘C’, ‘F’, ‘I’, ‘L’, and ‘T/O’ (turnover) as shown in bold font. This is due to Total Float being present to varying extents in non-critical activities where it could be expended without positive consequence to the overall schedule or to the critical activities on any parallel or adjacent paths. Accordingly, all calculations herein focus solely on the allocation of Contract Float to critical activities.

Table 1: Activity Cost Allocations

<table>
<thead>
<tr>
<th>Activity</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobilization</td>
<td>$50,000</td>
</tr>
<tr>
<td>A</td>
<td>$285,000</td>
</tr>
<tr>
<td>B</td>
<td>$145,000</td>
</tr>
<tr>
<td>C</td>
<td>$25,000</td>
</tr>
<tr>
<td>D</td>
<td>$210,000</td>
</tr>
<tr>
<td>E</td>
<td>$150,000</td>
</tr>
<tr>
<td>F</td>
<td>$195,000</td>
</tr>
<tr>
<td>G</td>
<td>$200,000</td>
</tr>
<tr>
<td>H</td>
<td>$100,000</td>
</tr>
<tr>
<td>I</td>
<td>$110,000</td>
</tr>
<tr>
<td>J</td>
<td>$250,000</td>
</tr>
<tr>
<td>K</td>
<td>$255,000</td>
</tr>
<tr>
<td>L</td>
<td>$310,000</td>
</tr>
<tr>
<td>M</td>
<td>$190,000</td>
</tr>
<tr>
<td>T/O</td>
<td>$25,000</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$2,500,000</td>
</tr>
</tbody>
</table>

Figure 1: Exemplar CPM Network Schedule Diagram

4.4.2 Calculation Normalization

The use of disparate elements in calculations (i.e. cost divided by duration) and partial arrays result in percentages that do not add to 100% of their respective category. They require a normalization such that their sum equals 100% and can be accurately compared to one another. To accomplish this, each element of the calculation will be multiplied by the reciprocal of the sum of all elements in the category.

For example, when calculating the square root-based Contract Float allocation to critical activities, the ‘Mob.’ activity duration of 7 days yields a square root of 2.65 ‘root-days’ that translates to 14.70% of the 18-day-long Contract Float duration. Multiplying the resulting percentage of each activity by 1/120% then normalizes the resulting percentages. Table 2 presents a partial depiction of the normalization process.
4.4.3 Measures of Dispersion

The measure of dispersion most germane to the posited allocation methodology is the standard deviation, as it quantifies the variation of data elements or their ‘scattering’ from the mean value. In similar fashion to the Penrose Law, it is a square root based derivative of the statistical variance. In this analysis a lower standard deviation indicates that the data points of the individually allocated Distributed Float (DF) – the allocated percent of Contract Float assigned to each critical activity – tend to be closer to the mean and closer to each other, thereby representing a more even or ‘fairer’ allocation across the critical activities.

<table>
<thead>
<tr>
<th>Table 2: Partial Exemplar Calculation – Duration-Based Normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Activity</strong></td>
</tr>
<tr>
<td>Mob.</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3: Partial Exemplar Outputs – Duration-Based Calculations and Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Activity</strong></td>
</tr>
<tr>
<td>Mob.</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
</tr>
</tbody>
</table>

4.5 Float Allocation Model via the Penrose Square Root Law

4.5.1 Distributed Float Calculations

Continuing the calculation description from above, the normalized duration square root-based array begins with the normalized percent of Contract Float (CF) – in this case 12.21% for the ‘Mob’ activity. This percentage is the basis for assignment of the respective Distributed Float (DF) – 12.21% of the 18-day-long Contract Float. For the ‘Mob’ critical activity this equates to a DF of 2.20 days. Tracing this as a single calculation from beginning to end, the duration square root-based ‘Mob.’ activity calculation follows the sequence depicted in Table 3.

4.5.2 Calculation Analysis

Viewing the dispersion of the Distributed Float as the measure for determining the applicability of cost or duration-based modeling and whether the application of the Penrose Square Root Law makes a material difference in the allocation of Contract Float to critical activities, it is apparent that using a calculation that is based on the square root of duration yields the smallest dispersion, i.e. the lowest standard deviation at 4.17%. Likewise, its array of allocation percentages maintains the tightest range from 7.99% to 19.58% (which is a 11.59% spread) versus from 2.91% to 36.05% (which a much larger 33.14% spread) for the nominal cost-based calculation that represents the greatest dispersion as listed in the results of Table 4.
Table 5 presents these results with the addition of the integer rounded Distributed Float resulting from each array. Of note is that despite having a tighter range and standard deviation than that of the nominal cost-based calculation, the nominal duration-based array produces an integer rounded Distributed Float total of 20 days, 2 days in excess of the 18 days of available Contract Float. This increased variation and resulting allocation in excess of the project parameters is a prima facie indication of and strengthens the Penrose precepts of the advantage of a square root based allocation model.

5. CONCLUSIONS

This research began has begun by linking decision-making models and voting practices with risk within network schedules and establishing the expectation of an improved methodology for allocating it among project participants for their individual activities. Through the literature it has confirmed that float is a means of mitigating risk in network schedules and has extended the current supposition that float, in particular Total Float, is a diminishing commodity, has no direct ownership or control, and is expended by the entity or activity of first want. Through the exemplar it has been demonstrated that the square root allocation model (to be determined on a case-by-case basis for its applicability to either cost-based or duration-based allocations) appears to be superior to that of the nominal duration or cost given its tighter dispersion. Current research is collecting time and cost data as well as contract and plan drawings from a real-world construction project to provide further validation of the new theory in form of a case study.

This research has contributed two definitions of new float types to the body of knowledge:

- **Contract Float**: The difference between the calculated duration of a network schedule per CPM and the contractually agreed completion time;
- **Distributed Float**: The allocation of Contract Float to the critical activities of a network schedule by a mathematical model. The total Distributed Float cannot exceed the available Contract Float.

More importantly, this research concludes that Contract Float allocation is singularly applicable to critical activities, but not to all network schedule participants. By definition, critical activities are traditionally afforded no float within the CPM or construction network schedule, thereby relegating Total Float to the non-critical activities and rendering moot the need for allocation (to non-critical activities) of float residing ‘outside’ the network duration. That is, Free Float (also called Activity Float) remains completely undisturbed by the allocation of Contract Float to critical path activities. In fact, non-critical activities with or without the presence of Activity Float may in certain instances – when occurring parallel to and ending coincident with that of a critical activity, benefit from the allocation of Contract Float in that these non-critical activities gain the potential for concurrent expenditure of Contract Float, should it be consumed.
6. FUTURE RESEARCH

This research represents the first element in the three-part research on risk location, quantification, and pricing to mitigate and reduce its negative impacts. The second element will address its predominant location within network schedules. The third element will be the specific pricing for a comprehensive float trading model. Similar to this research, conceptual analogies currently in existence or practical use will be explored. It is expected that a working predictive modeling mechanism will result from the analysis of risk location in network schedules toward a market wherein participants can exchange and trade it as needed.

7. REFERENCES