Modeling Resource Profiles with Singularity Functions

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Abstract
Resources are productive agents on construction projects and are therefore of special interest in project planning. Algorithms use heuristic or evolutionary approaches to (a) allocate resources underneath an availability ceiling to determine the minimum possible project duration or (b) leveling resources of noncritical activities within the fixed project duration toward a smooth profile of an even workflow. However, adding one or several types of resources in histogram bins is scale-dependent, disconnected from the underlying schedule, and computationally inefficient, as any change requires a recalculation. This paper therefore describes how such profiles can be modeled with singularity functions, which have been applied to linear scheduling, as cumulative additions over time. An example demonstrates how the activities and their resources remain coupled during an optimization procedure, e.g. resource leveling. Resources are extracted directly from productivities of their activities. The new approach also allows defining mathematical expressions for specific desirable or available profiles.

Challenges of Resource Planning and Control
Resource is a broad term that can include materials, labor, and equipment that are directly involved in the productive activities on a construction project, as well as a wide range of less tangible items, e.g. permits, access, information, and money that function as conditions or enabling factors for actual execution. As used in this paper it refers to any specialized labor, e.g. crafts; other types are rare in the literature (Smith-Daniels and Aquilano 1984). Labor combined with heavy equipment is called a crew. Optimizing the use of limited resources is an important objective of project managers. Main approaches are allocation (Lambourn 1963) that determines the project duration by placing resources of activities under an availability ceiling and leveling (Galbreath 1965) that shift resources within float from peaks into valleys in their histogram. Resource leveling (Harris 1990), toward which the new model described in this paper is applied, is an NP-complete (Son and Skibniewski 1999) problem of algorithmically ‘smoothening’ a resource profile to a rectangle to create an uninterrupted progress of the project and minimize costs of hiring, training, and firing plus avoid any decreased morale among the laborers from a potentially high turnover. Besides construction
engineering and management (e.g. Kandil and El-Rayes 2006), operations research and management science studies (Brucker et al. 1999, Herroelen et al. 1998), rarely cross-cited in construction research, also examined the similarly complex resource-constrained project scheduling problem, e.g. with standard problem sets (Kolisch and Sprecher 1996). Resource-constrained schedules are NP-hard (Błazewicz et al. 1983).

Previous Limitations for Linear Scheduling
Several studies applied resource leveling to linear scheduling. Mattila and Abraham (1998) incurred rounding errors in integer linear programming of delays and lower productivity. Yang and Ioannou (2001) used a pull approach to avoid idleness. Yang and Chang (2005) modeled probabilistic durations but again allowed only constant productivity. Yen (2005) worked on combining resource allocation and leveling using heuristics and meta-heuristics, but noted its one-directional model, inefficient model of variable productivity, and large number of variables. Georgy (2008) used a similar genetic algorithm to minimize daily changes. None of the approaches could derive its resource model directly from its linear schedule for an integrated optimization. Their different limitations can be overcome by applying the flexible singularity functions.

Basic Operator of Singularity Functions
The basic operator that composes the individual terms that make up each singularity function is written as a case distinction as per Eq. 1. The pointed brackets were first used by Wittrick (1965), while singularity functions had been independently defined by Föppl (1927) and Macaulay (1919) for structural analysis, e.g. of beams under a combination of vertical loads. The operator is zero (inactive) for all x-values smaller than the cutoff value $a$. Beginning at $a$, it gives the value of evaluating the bracketed term normally and remains active until infinity in the positive direction of x. The x-axis is horizontal and the y-axis is vertical. The exponent $n$ determines the overall behavior, e.g. constant, linear, quadratic, etc. and the factor $b$ provides its scaling, e.g. for the height of a constant, the slope of a linear growth, or the shape of a parabola.

$$b \cdot (x-a)^n = \begin{cases} 0 & \text{for } x < a \\ s \cdot (x-a)^n & \text{for } x \geq a \end{cases}$$

Eq. 1

Additive Superposition and Simplification
Basic operators are assembled into a complete singularity function based on ranges of behavior in the variable of interest, $y(x)$. Each change in the behavior is modeled by adding a new term as per Eq. 1. A singularity function thus can contain an unlimited number of terms that cumulatively describe the entire gradient of $y(x)$ for any two-dimensional shape as long as it qualifies as a true function, i.e. only one output value $y(x)$, which can be zero, exists for each input x-value. Note that discontinuities, e.g. vertical steps, can be modeled, so that the singularity (i.e. points of change) functions occasionally are also called discontinuity functions. This important ability directly enables modeling the components and analyzing the resource profile in this paper. Superposition refers to the cumulative nature of singularity functions. Keeping the components separated into ranges, i.e. segments, and within them into the behaviors of different exponents creates unique flexibility to model complex shapes. Extensions
to Eq. 1, e.g. to allow behavior depending on $x$ itself or logarithms, are possible, but have not been described in the literature and will be investigated in future research. Sorting all terms in a singularity function by their cutoffs $a$ and within that by their exponent $n$ increases the clarity, especially of manual calculations. It also facilitates simplifying any terms whose factors $b$ can be added if their $a$ and $n$ are both identical. Simplification also reduces the required storage for computer calculations, wherein an increasing number of values from active terms would be kept for growing $x$-values.

**Differentiation and Integration of Basic Operator**

Singularity functions have a fundamental advantage over other methods for modeling any discontinuous shapes, e.g. vectorization (Russell and Caselton 1988). As per Eqs. 2 and 3, they can be differentiated to calculate its rate of change or integrated for the area underneath the function, e.g. the total resource consumption of a resource profile.

$$\frac{d}{dx} b \cdot (x-a)^n = n \cdot b \cdot (x-a)^{n-1}$$  \hspace{1cm} \text{Eq. 2}

$$\int b \cdot (x-a)^n = \frac{b}{n+1} \cdot (x-a)^{n+1} + C$$  \hspace{1cm} \text{Eq. 3}

**Singularity Functions in Linear Scheduling**

Lucko (2009) presented a novel use of singularity functions to model an entire linear schedule, which is particularly suited for any projects with a linear geometry and/or repetitive operations (Mattila and Park 2003). Its criticality analysis had six steps:

1. Writing all activities and their buffers as singularity functions with a zero intercept.
2. Sequentially stacking activities and buffers alternatingly with a conservative finish-to-start relationship, i.e. the maximum of a predecessor is the intercept of a successor.
3. Calculating pairwise differences between adjacent buffer and activity equations.
4. Differentiating these difference equations to find their minima, i.e. critical points.
5. Consolidating the schedule to its minimum possible duration under consideration of all constraints by deducting the minimum differences and rewriting all intercepts.
6. Determining the continuous critical path from the critical points. An analysis of the different float types (Lucko and Peña Orozco 2009) can be performed subsequently.

**Modeling Individual Resource Consumption**

Each activity consumes resources, e.g. laborers, to produce a certain physical output. Assuming that its resource rate $r$, which can be measured e.g. in number of laborers, remains constant over time for an activity, Eq. 4 models this individual consumption.

$$r(y) = r \cdot (y-a_s)^0 - r \cdot (y-a_f)^0$$  \hspace{1cm} \text{Eq. 4}

where $r(y)$ is the resource consumption, $y$ is the time on the horizontal axis, $r$ is the individual resource rate of an activity, $a_s$ is the start time of said activity, and $a_f$ is its finish time. More complex consumption patterns could thus be modeled additively.
Under this new approach each activity contributes a horizontal strip to the resource profile. Prior studies, e.g. Georgy (2008), Mattila and Abraham (1998), and Galbreath (1965), all used vertical strips as histogram bins for each time unit, e.g. workdays. An extra algorithm step of adding daily aggregated resources (Russell and Dubey 1995) was required to calculate the so-called improvement factor (Hiyassat 2000, Martínez and Ioannou 1993) for each possible time shift of the noncritical activities within the profile. The new model does not split the resources from their activities. It facilitates an optimization, e.g. to level the profile by shifting noncritical activities or segments thereof, which only modifies the particular cutoff value \( a \). Adding the expressions of Eq. 4 for all activities gives the complete resource profile for the construction project.

**Modeling Measure of Resource Levelness**

The first moment of area, used to calculate the centroid of complex shapes and in its extension to the second moment of area for the bending stiffness of cross-sections in structural engineering, is a generally used measure by which to compare the different possible constellations of a resource profile and select the one with minimum moment (Harris 1990), i.e. an increased levelness. It is the area of a regular geometric shape multiplied by the length of the lever from its center of gravity to the axis around which a rotation is imagined, in this case the horizontal \( y \)-axis. Complex shapes can be decomposed into simpler ones, e.g. rectangles. Note that each activity adds not one but two horizontal strips to the resource profile and accordingly also to the moment. One strip beginning at its start is positive; one beginning at its finish is negative.

Modeling the moment must correctly consider both the area and lever of an activity’s contribution to the resource profile. The area, i.e. resource consumption, of an activity is always constant, but could be ‘split’ into segments at different heights when adding it onto an existing uneven profile. Another complication is caused by any concurrent activities whose levers cannot be determined from their resource rate alone. Using the singularity function that expresses the entire resource profile provides the solution. Eq. 5 models the moment of a resource profile as the sum over the products of each newly added strip multiplied by the cumulative total height (including said strip) of the profile up to that time but minus half the height of said strip to reach its center of gravity. Note again that each activity contributes two strips to the total summation.

\[
M = \sum_{i=1}^{2} \left( r_i \cdot \left( \sum_{j=1}^{k} r_j \right) \right) - 0.5 \cdot r_i \]

where \( M \) is the first moment of area, \( i \) is a running index of all activities that are sorted by their start time up to their total count \( k \), \( j \) is a running index within \( i \), \( r \) is the individual resource rate of an activity, and \( a \) is a start or finish time of said activity. The pointed brackets in Eq. 5 calculate the length of the strip from its start time until the maximum finish time \( y_k \), i.e. the rightmost edge of the entire resource profile. The summation nested inside Eq. 5 can also be rewritten as a recursive expression of \( r(y) \).

**Modeling Changes in Resource Profile**

Two possible ways exist by which a complete activity changes its contribution to the profile while maintaining continuity, shifts and resource changes. They modify Eq. 4
to Eq. 6 by adding a shift \( s \) to its cutoff \( a \) and changing the resource rate \( r \). The total ‘area’ \( R \), which can be measured e.g. in labor-days, is maintained via the activity duration. Additionally, these changes can affect individual activity segments. Shifting them creates interruptability. Even more complexity is introduced if intact activities are permitted to become interruptible at any point in time (Ioannou and Srisuwanrat 2007) and are possibly subject to additional calendar-related constraints, both which is not considered in this analysis example and will be investigated in future research.

\[
 r(y)=r \cdot (y-(a_s+s))^0 - r \cdot (y-(a_s+(R_{var}/r)+s))^0
\]

Eq. 6

where \( s \) is the shift to the cutoff for the start of the activity and \( R_{var} \) is the variable resource content of said noncritical activity segment as the product of its duration multiplied by its resource rate \( r \), i.e. the area of its contribution to the resource profile.

**Resource Example**

For brevity and due to this paper focusing on new modeling concepts, only resource leveling is presented in the following example. The related resource allocation could be performed with analogous concepts and will be investigated in future research. Originally presented by Mattila and Abraham (1998) for a real road building project, the example of the linear schedule was analyzed for critical segments as shown on the left side of Figure 1. Table 1 lists its parameters as needed for the resource leveling.

**Table 1: Activity List and Critical Ranges**

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Activity Duration</th>
<th>Work Amount</th>
<th>Start Time</th>
<th>Finish Time</th>
<th>Start Amount</th>
<th>Critical Time</th>
<th>Critical Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Full</td>
<td>5</td>
<td>50</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0 to 2</td>
<td>0 to 20</td>
</tr>
<tr>
<td>B</td>
<td>Bar</td>
<td>3</td>
<td>N/A</td>
<td>0</td>
<td>3</td>
<td>42</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>C</td>
<td>Full</td>
<td>15</td>
<td>50</td>
<td>2</td>
<td>17</td>
<td>0</td>
<td>2 to 17</td>
<td>0 to 50</td>
</tr>
<tr>
<td>D</td>
<td>Block</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>13</td>
<td>8</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>Full</td>
<td>8</td>
<td>50</td>
<td>12</td>
<td>20</td>
<td>0</td>
<td>17 to 20</td>
<td>31.25 to 50</td>
</tr>
<tr>
<td>F</td>
<td>Partial</td>
<td>2</td>
<td>20</td>
<td>20</td>
<td>22</td>
<td>30</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>G</td>
<td>Full</td>
<td>13</td>
<td>50</td>
<td>20</td>
<td>33</td>
<td>0</td>
<td>20 to 33</td>
<td>0 to 50</td>
</tr>
<tr>
<td>H</td>
<td>Full</td>
<td>5</td>
<td>50</td>
<td>30</td>
<td>35</td>
<td>0</td>
<td>33 to 35</td>
<td>30 to 50</td>
</tr>
<tr>
<td>I</td>
<td>Full</td>
<td>3</td>
<td>50</td>
<td>35</td>
<td>38</td>
<td>0</td>
<td>35 to 38</td>
<td>0 to 50</td>
</tr>
</tbody>
</table>

Boldface values in Table 1 indicate critical points, i.e. where a change from critical to noncritical occurs or vice versa. Permutations of activities or segments thereof are shown as dotted lines and straight or curved arrows in Figure 1 to indicate different possible contributions to the resource profile. Table 2 lists the float that enables time shifts and resource rates to vary for individual activities. The resource rates count the shared type of equipment, trucks. Underlined values indicate the initial configuration of each activity as shown in solid lines in Figure 1. The areas \( R \) in the profile have the unit of resources multiplied by duration, i.e. truck-days. The work amount in this road building example was measured as length in surveying stations \([30.48 \text{ m} \approx 100 \text{ ft}]\).
Figure 1: Linear Schedule (Mattila and Abraham 1998) and Resource Profile

Table 2: Float, Time Shifts, and Resource Rates

<table>
<thead>
<tr>
<th>Name</th>
<th>Slope</th>
<th>Float</th>
<th>Shift</th>
<th>Resources</th>
<th>Fixed Area</th>
<th>Variable Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5/50</td>
<td>Partial, late</td>
<td>0, 2, 4</td>
<td>2, 3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>B</td>
<td>N/A</td>
<td>Fully</td>
<td>0, 5, 10</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>C</td>
<td>15/50</td>
<td>Fixed</td>
<td>N/A</td>
<td>N/A</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>N/A</td>
<td>Fully</td>
<td>0, -1</td>
<td>Constant 8</td>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>E</td>
<td>8/50</td>
<td>Partial, early</td>
<td>Constant 0</td>
<td>4, 5</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>F</td>
<td>5/50</td>
<td>Fully</td>
<td>0, 1, 2, 3</td>
<td>N/A</td>
<td>78</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>13/50</td>
<td>Fixed</td>
<td>N/A</td>
<td>N/A</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>5/50</td>
<td>Partial, early</td>
<td>0, -3</td>
<td>4, 5, 6, 8</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>I</td>
<td>3/50</td>
<td>Fixed</td>
<td>N/A</td>
<td>N/A</td>
<td>24</td>
<td>0</td>
</tr>
</tbody>
</table>

Fixed Profile of Critical Activity Segments

All critical activities give contributions that remain fixed during the resource leveling and are shaded on the right side of Figure 1. Eq. 7 models this consumption pattern by adding the contributions from all critical activity segments as per Eq. 4 and Table 1.
Note that the resource profile on the right side of Figure 1 is rotated by 90 degrees. This orientation enabled the y-variable of its linear schedule to be minimized (Lucko 2009). It would be beneficial to transpose the time axis of the profile, or histogram, to become horizontal to visualize optimizing the variable of interest r. However, it is not required, as singularity functions are independent of their graphical representation. It is possible to directly extract r from the known unit productivity of each activity in the linear schedule once the transposition or ‘axis flip’ has been performed so that, unlike in Figure 1, the slope of each activity curve is proportional to its productivity.

Variable Contributions from Noncritical Activity Segments
Noncritical activities have float that can be consumed by shifts s or by changed r that extend an activity’s duration. Either parameter may vary during the resource leveling. Eqs. 8 through 13 apply Eq. 6 to all of the noncritical segments of late A, B, D, early E, F, and early H based on their possible integer values for s and r as per Table 2. Eq. 6 is modified to model the change in the start – not finish – time of the early E and H.

\[
\begin{align*}
    r(y)_A &= r_A \cdot \langle y-(2+s_A) \rangle^0 - r_A \cdot \langle y-(2+9/r_A+s_A) \rangle^0 \\
    r(y)_B &= 1 \cdot \langle y-(0+s_B) \rangle^0 - 1 \cdot \langle y-(0+3/1+s_B) \rangle^0 \\
    r(y)_D &= 8 \cdot \langle y-(10+s_D) \rangle^0 - 8 \cdot \langle y-(10+24/8+s_D) \rangle^0 \\
    r(y)_E &= r_E \cdot \langle y-(17-25/r_E+0) \rangle^0 - r_E \cdot \langle y-(17+0) \rangle^0 \\
    r(y)_F &= 2 \cdot \langle y-(20+s_F) \rangle^0 - 2 \cdot \langle y-(20+4/2+s_F) \rangle^0 \\
    r(y)_H &= r_H \cdot \langle y-(33-24/r_H+s_H) \rangle^0 - r_H \cdot \langle y-(33+s_H) \rangle^0
\end{align*}
\]

Eq. 7

Eq. 8

Eq. 9

Eq. 10

Eq. 11

Eq. 12

Eq. 13

Moment from Critical and Noncritical Activity Segments
Resource rates r and shifts s, i.e. interruptions between activity segments, can vary in integer counts as per Table 2. These given values are assumed to not violate buffers between activities. Performing the resource leveling thus seeks a minimized moment among all possible permutations of \(s_A = \{0, 2, 4\}, r_A = \{2, 3\}\), \(s_B = \{0, 5, 10\}\), \(s_D = \{0, -1\}\), \(r_E = \{4, 5\}\), \(s_F = \{0, 1, 2, 3\}\), \(s_H = \{0, -3\}\), and \(r_H = \{4, 5, 6, 8\}\), which gives \(3 \cdot 2 \cdot 3 \cdot 2 \cdot 2 \cdot 4 \cdot 2 \cdot 4 = 2,304\) possible permutations; larger examples can have millions of permutations. Using Eqs. 7 through 13 it is now possible to compose Eq. 14 for the moment based on Eq. 5. All these change terms are sorted, simplified, and extended by the length of the strip from its start until the maximum y-value and the appropriate lever. However, a correction variable becomes necessary for the case that a positive
and a negative strip have the same cutoff value \( a \). Adding them would cancel out their contributions to the moment, which would give too short a lever. Correcting this issue is accomplished by introducing an infinitesimal duration \( \varepsilon \) that is subtracted from the cutoff values of all strips that are a ‘downward’ step in the resource profile, i.e. those strips with a negative sign. Alternatively, it could be added to the cutoff value of all the ‘upward’ steps, which would also keep downward and upward steps separate. This prevents any simplification toward an incorrect moment. The value of \( \varepsilon \) is set to be extremely small, e.g. \( 1 \cdot 10^{-6} \), to not influence any significant digits of the moment.

\[
M = 3 \cdot \langle 38 \rangle \cdot \lbrack r(0) - 1.5 \rbrack + 1 \cdot \langle 38 - s_B \rangle \cdot [r(0 + s_B) - 0.5] + 1 \cdot \langle 36 \rangle \cdot \lbrack r(2) - 0.5 \rbrack
+ r_A \cdot \langle 36 - s_A \rangle \cdot [r(2 + s_A) - 0.5 \cdot r_A] - 1 \cdot \langle 35 - s_B \rangle \cdot \lbrack r(3 + s_B - \varepsilon) - 0.5 \rbrack
- r_A \cdot \langle 36 - s_A - (9/r_A) \rangle \cdot [r(2 + s_A + (9/r_A) - \varepsilon) - 0.5 \cdot r_A] + 8 \cdot \langle 28 - s_D \rangle \cdot [r(10 + s_D) - 4]
+ r_E \cdot \langle 21 + (25/r_E) \rangle \cdot [r(17 - (25/r_E)) - 0.5 \cdot r_E] - 8 \cdot \langle 25 - s_D \rangle \cdot [r(13 + s_D - \varepsilon) - 4]
- r_E \cdot \langle 21 \rangle \cdot \lbrack r(17 - \varepsilon) - 0.5 \cdot r_E \rbrack + 1 \cdot \langle 21 \rangle \cdot \lbrack r(17) - 0.5 \rbrack
+ 1 \cdot \langle 18 \rangle \cdot [r(20) - 0.5] + 2 \cdot \langle 18 - s_f \rangle \cdot [r(20 + s_f) - 1] - 2 \cdot \langle 16 - s_f \rangle \cdot [r(22 + s_f - \varepsilon) - 1]
+ r_H \cdot \langle 5 - s_H + (24/r_H) \rangle \cdot \lbrack r(33 + s_H - (24/r_H) - 0.5 \cdot r_H \rbrack - r_H \cdot \langle 5 - s_H \rangle \cdot \lbrack r(33 + s_H - \varepsilon) - 0.5 \cdot r_H \rbrack + 2 \cdot \langle 5 \rangle \cdot \lbrack r(33) - 1 \rbrack - 8 \cdot \langle 0 \rangle \cdot \lbrack r(38 - \varepsilon) - 4 \rbrack
\]

Eq. 14

All strips end at the maximum \( y \)-value of 38 days. Eq. 14 can be evaluated for any permutations of \( s \) and \( r \) as per Table 2, e.g. for the initial permutation with zero shifts and the maximum number of trucks (i.e. \( s_A = 0, r_A = 3, s_B = 0, s_D = 0, r_E = 5, s_F = 0, s_H = 0, \) and \( r_H = 8 \)). All possible simplifications must be performed and for clarity are underlined: \( M = (3 + 1) \cdot 38 \cdot [4 - 1.5 - 0.5] + (1 + 3) \cdot 36 \cdot [8 - 0.5 - 1.5] - 1 \cdot 35 \cdot [8 - 0.5 - 3 \cdot 33 \cdot [7 - 1.5] + 8 \cdot 28 \cdot [12 - 4] + 5 \cdot 26 \cdot [17 - 2.5] - 8 \cdot 25 \cdot [17 - 4] - 5 \cdot 21 \cdot [9 - 2.5] + 1 \cdot 21 \cdot [5 - 0.5] + [1 + 2] \cdot 18 \cdot [8 - 0.5 - 1] - 2 \cdot 16 \cdot [8 - 1] + 8 \cdot 8 \cdot [14 - 4] - 8 \cdot 5 \cdot [14 - 4] + 2 \cdot 5 \cdot [8 - 1] - 8 \cdot 0 \cdot 0 = 1,287 \) truck-days times trucks. This value is validated by a manual calculation of the right side of Figure 1 and found to be correct.

**Conclusions**

The new model can be efficiently used in any type of optimization procedure toward resource leveling, using e.g. a heuristic or evolutionary algorithm that is applied to the parameters \( s \) and \( r \). For brevity, details of such solution will be described under future research. The new model has been implemented in a computer application and can be used in conjunction with any schedule, but is especially suited for linear scheduling. It is planned to integrate the output of the model with a genetic algorithm, whose optimized configuration for the leveled resource profile will be validated against the results provided by Mattila and Abraham (1998). On the input side, the model can be integrated with an existing software tool for performing a criticality and float analysis of linear schedules with singularity functions. Overall, this model thus provides a link between different analytical functions of project management. It opens an analytical avenue for modeling non-linear, discontinuous characteristics of complex phenomena as e.g. encountered in resource planning in an integrated, coherent, and exact manner.
**Future Research**

The model for modeling and optimizing resource profiles that has been presented in this paper is a stepping stone toward research on integrated project management. The resources were derived directly from their underlying activities in the linear schedule, thus maintaining a mathematical link between the schedule and its resource profile. Future research will extend the model and its analytical capabilities toward different behaviors of resources within activities, e.g. to model a learning effect, using several different types of resources with their respective availability and cost constraints and different weights or priorities, and investigating its application to resource allocation, where different availability ceilings can also be modeled with singularity functions.

**References**


